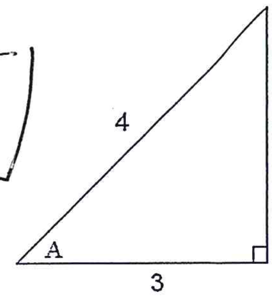
Worksheet 5.5 **Precalculus** **Name** ______________________________
Period _____

1) Use the figure shown in quadrant I to find the exact value of the trigonometric functions.

$3^2 + b^2 = 4^2$
$b^2 = 16 - 9$
$b^2 = 7$
$b = \sqrt{7}$



$(3\sqrt{7})^2 + a^2 = 8^2$
$63 + a^2 = 64$
$a^2 = 1$
$a = 1$

A) $\sin A = \boxed{\frac{\sqrt{7}}{4}}$

B) $\cos A = \boxed{\frac{3}{4}}$

C) $\sin 2A =$ $2\sin A\cos A$
$= \cancel{2} \cdot \frac{\sqrt{7}}{\cancel{4}_2} \cdot \frac{3}{4} =$ $\boxed{\frac{3\sqrt{7}}{8} \text{ or } \frac{3}{8}\sqrt{7}}$

D) $\cos 2A =$ $\cos^2 A - \sin^2 A$
$\left(\frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2 =$ $\frac{9 - \cancel{49}\,7}{16} = \cancel{\frac{-40}{16} = \frac{-5}{2}}$ $\frac{2}{16} = \boxed{\frac{1}{8}}$

E) $\sec 2A = \boxed{8}$

F) $\tan \frac{A}{2} =$ $\frac{1 - \cos A}{\sin A} = \frac{1 - \frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{1}{\sqrt{7}} = \boxed{\frac{\sqrt{7}}{7}}$
$= \frac{\frac{1}{4}}{\frac{\sqrt{7}}{4}} =$

2) Let $\sec\theta = -\frac{13}{5}$ where $\frac{\pi}{2} < \theta < \pi$. Find the exact values of the trigonometric functions.

$\cos = -\frac{5}{13}$ $\sin\theta = \frac{12}{13}$ $\tan\theta = -12/5$ 12 13 -5 ✓ $\theta/2$ in QI
12 13
2θ in Q3

A) $\cos 2\theta =$ $\cos^2\theta - \sin^2\theta$
$\left(\frac{-5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$
$\frac{25}{169} - \frac{144}{169} = \boxed{\frac{-119}{169}}$

B) $\sin\frac{\theta}{2} =$ $\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1+\frac{5}{13}}{2}} = \sqrt{\frac{\frac{13}{13}+\frac{5}{13}}{2}} = \sqrt{\frac{\frac{18}{13}}{2}} = \sqrt{\frac{18}{26}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}$
$= \boxed{\frac{3\sqrt{13}}{13}}$

QI so + quadrant I, so positive

C) $\tan 2\theta =$ $\frac{\sin 2\theta}{\cos 2\theta}$ $\sin 2\theta = 2\cos\theta\sin\theta$
$2 \cdot \frac{-5}{13} \cdot \frac{12}{13} = \frac{-120}{169}$

$\frac{\frac{-120}{169}}{\frac{-119}{169}} = \boxed{\frac{120}{119}}$

TURN this paper OVER for the rest of the worksheet

3) Find the solutions to the equation that are in the interval $[0, 2\pi)$.

$\sin 2\theta = -2\cos\theta$

$\sin 2\theta + 2\cos\theta = 0$

$2\sin\theta\cos\theta + 2\cos\theta = 0$

$2\cos\theta(\sin\theta + 1) = 0$

$2\cos\theta = 0$ $\qquad$ $\sin\theta = -1$

$\cos\theta = 0$

$\boxed{\pi/2}$

$\boxed{3\pi/2}$ $\qquad$ same

4) Rewrite the expression in terms of the first power of the cosine.

A) $\sin^2\theta\cos^2\theta =$

$\dfrac{(1-\cos 2x)}{2} \cdot \dfrac{(1+\cos 2x)}{2}$

$\dfrac{1-\cos^2(2x)}{4}$

$\dfrac{1-\left(\dfrac{1+\cos 2(2x)}{2}\right)}{4}$

$\dfrac{\frac{2}{2} - \frac{1+\cos 4x}{2}}{4} = \dfrac{\frac{1+\cos 4x}{2}}{4}$

$\dfrac{2}{8} - \left(\dfrac{1+\cos 4x}{8}\right) = \dfrac{1}{8}(1-\cos 4x)$

B) $\sin^4\theta =$

$(\sin^2\theta)^2$

$\left(\dfrac{1-\cos 2x}{2}\right)^2$

$\dfrac{1-2\cos 2x+\cos^2(2x)}{4}$

$\dfrac{1-2\cos 2x + \frac{1+\cos 2(2x)}{2}}{4}$

$\dfrac{1-2\cos 2x}{4} + \dfrac{1+\cos 4x}{8}$

$\dfrac{1}{4} - \dfrac{2\cos 2x}{4} + \dfrac{1}{8} + \dfrac{\cos 4x}{8}$

$\dfrac{3}{8} - \dfrac{\cos 2x}{2} + \dfrac{\cos 4x}{8}$

C) $\cos^4\theta =$

$(\cos^2\theta)^2$

$\left(\dfrac{1+\cos 2x}{2}\right)^2$

$\dfrac{1+2\cos 2x+\cos^2(2x)}{4}$

$\dfrac{1}{4} + \dfrac{2\cos 2x}{4} + \dfrac{\frac{1+\cos 2(2x)}{2}}{4}$

$\dfrac{1}{4} + \dfrac{\cos 2x}{2} + \dfrac{1+\cos(4x)}{8}$

$\dfrac{1}{4} + \dfrac{1}{8} + \dfrac{\cos 2x}{2} + \dfrac{\cos 4x}{8}$

$\dfrac{3}{8} + \dfrac{\cos 2x}{2} + \dfrac{\cos 4x}{8}$

You answers just need to be equivalent.