b. Possible construction method: Construct golden rectangle *ABCD* following the method from 27a. For square *AEFD,* locate $\overline{EF}$ by constructing circle *A* and circle *D* each with radius *AD*. Repeat the process of cutting off squares as often as desired. For the golden spiral from point *D* to point *E,* construct circle *F* with radius *EF;* select point *D,* point *E,* and circle *F,* and choose **Arc On Circle** from the Construct menu.

DEVELOPING MATHEMATICAL REASONING

Answer	○	○	●	●	
Guess #4	●	●	○	○	⊖⊖⊖⊖
Guess #3	○	○	●	○	⊕⊖
Guess #2	○	●	○	●	⊕⊖⊖
Guess #1	○	○	○	○	⊖

From Guess #4 there are no yellow discs in the final arrangement.

From Guess #4, red is not in columns 1 or 2, green is not in column 3, orange is not in column 4.

From Guess #1 and #4, green is in column 1.

Orange is not in columns 1 or four, and there is no yellow disc, so by Guess #3 red is in column 3.

Orange is not in column 4, green is in column 1, and red is in column 3, orange is in column 2, therefore, red must also be in column 4.

CHAPTER 7 REVIEW

EXERCISES

1. False. Equiangular means that the angles are congruent. Sides could be any length.

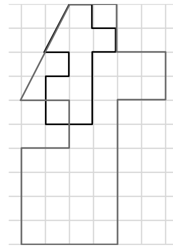
2. True. A dilation by a scale of 1 is the dilation rule $(x, y) \rightarrow (1x, 1y)$, or $(x, y) \rightarrow (x, y)$. So the figures are the same, and are congruent.

3. True. When a figure is dilated by a scale factor of 3, the side lengths of the image are three times the side lengths of the original figure. The sum of the lengths, the perimeter of the figure, would be three times larger.

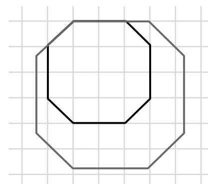
4. False. The dilation rule should be $(x, y) \rightarrow \left(\frac{5}{3}(x-7), \frac{5}{3}(y-10)\right)$.

5. True. You can use set up a similar triangle situation with known values and find the corresponding height of the object.

6. Answers will vary. Dilation of 2.

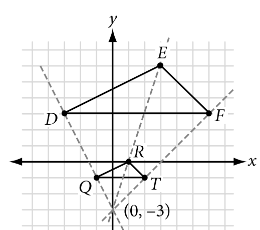


7. Answers will vary. Dilation of $\frac{3}{2}$.



8. $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y - 2\right)$; center of dilation $(0, -3)$.

The coordinates of the vertices of $\triangle DEF$ are $D(-3, 3)$, $E(3, 6)$, and $F(6, 3)$. The coordinates of the vertices of $\triangle QRT$ are $Q(-1, -1)$, $R(1, 0)$, and $T(2, -1)$. The *x*-coordinates of the vertices of $\triangle QRT$ are the *x*-coordinates of $\triangle DEF$ times one-third. The *y*-coordinates of the vertices of $\triangle QRT$ are the *y*-coordinates of $\triangle DEF$ times one-third and then subtract 2. So the dilation rule is $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y - 2\right)$. To find the center of dilation, find the point of concurrency of the lines connecting corresponding vertices (see graph below), $(0, -3)$.



9. Quadrilateral *MNRS* ≅ quadrilateral *EFGH* and quadrilateral *BCDA* ≅ quadrilateral *QMOP* because they have congruent sides and angles. Quadrilateral *MNRS* ~ quadrilateral *BCDA*, quadrilateral *MNRS* ~ quadrilateral *QMOP*, quadrilateral *EFGH* ~ quadrilateral *BCDA*, and quadrilateral *EFGH* ~ quadrilateral *QMOP* because their sides are proportional and angles congruent.

10. They are not similar. The ratio of the lengths of the triangles, $\frac{LB}{LS} = \frac{5}{10}$ and $\frac{LD}{LZ} = \frac{5}{9}$, are not equal, therefore, they are not similar.

11. $z = 4.1\bar{6}$, or $4\frac{1}{6}$

12. $w = 6$ cm, $x = 4.5$ cm, $y = 7.5$ cm, $z = 3$ cm. $ABCDE \sim FGHIJ$, so corresponding sides are proportional.

$$\frac{AB}{FG} = \frac{BC}{GH} = \frac{CD}{HI} = \frac{DE}{IJ} = \frac{AE}{FJ}$$
$$\frac{6}{9} = \frac{4}{w} = \frac{3}{x} = \frac{5}{y} = \frac{2}{z}$$
$$\frac{2}{3} = \frac{4}{w} = \frac{3}{x} = \frac{5}{y} = \frac{2}{z}$$

From $\frac{4}{w} = \frac{2}{3}$, $w = 6$ cm.

From $\frac{3}{x} = \frac{2}{3}$, $x = \frac{9}{2} = 4.5$ cm.

From $\frac{5}{y} = \frac{2}{3}$, $y = \frac{15}{2} = 7.5$ cm.

From $\frac{2}{z} = \frac{2}{3}$, $z = 3$ cm.

13. $x = 4\frac{1}{6}$ cm, $y = 7\frac{1}{2}$ cm. $\triangle ABC \sim \triangle DBA$, so corresponding sides are proportional.

$$\frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA}$$
$$\frac{5}{x} = \frac{6}{5} = \frac{9}{y}$$

From $\frac{5}{x} = \frac{6}{5}$, $6x = 25$, and $x = \frac{25}{6} = 4\frac{1}{6}$ cm.

From $\frac{6}{5} = \frac{9}{y}$, $6y = 45$, and $y = \frac{45}{6} = 7\frac{1}{2}$ cm.

14. $\triangle TUV \sim \triangle XWV$. By Alternate Interior Angles, $\angle T \cong \angle X$ and $\angle U \cong \angle W$, so the triangles are similar by AA Similarity.

15. Yes. According to the markings the angles are congruent and corresponding sides are proportional.

16. 21 feet. Set up the proportion $\frac{6}{2.5} = \frac{x}{8.75}$. Solve for x. $2.5x = 6 \cdot 8.75$. $x = 21$ ft

17. 13 ft 2 in. First change 5 ft 8 in. to $5\frac{2}{3}$ ft, 11 ft 3 in. to $11\frac{1}{4}$ ft, and 8 ft 6 in. to $8\frac{1}{2}$ ft. Let h represent the height of the tree. Use the similar right triangles to write a proportion; then solve the proportion to find h.

$$\frac{h}{11\frac{1}{4} + 8\frac{1}{2}} = \frac{5\frac{2}{3}}{8\frac{1}{2}}$$

$$\frac{h}{19\frac{3}{4}} = \frac{5\frac{2}{3}}{8\frac{1}{2}}$$

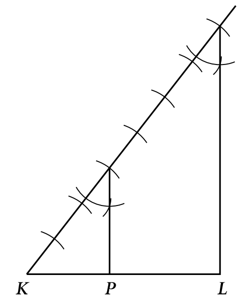
$$\frac{h}{\frac{79}{4}} = \frac{\frac{17}{3}}{\frac{17}{2}} = \frac{17}{3} \cdot \frac{2}{17} = \frac{2}{3}$$

$$\frac{h}{\frac{79}{4}} = \frac{2}{3}$$

$$h = \frac{79}{4} \cdot \frac{2}{3} = \frac{158}{12} = 13\frac{1}{6} \text{ ft} = 13 \text{ ft } 2 \text{ in.}$$

18. It would still be a 20° angle. A dilation doesn't change angle measures.

19. The method from Lesson 11.7, Example C can be used to divide $\overline{KL}$ into seven equal lengths. However, once you have constructed the segment that connects the point where the seventh arc intersects the ray to L, it is necessary only to connect the point where the third of the seven arcs intersects the ray with $\overline{KL}$ by constructing a parallel line. Let P be the point where this line intersects $\overline{KL}$. Then $KP = \frac{3}{7}(KL)$ and $PL = \frac{4}{7}(KL)$, so $\frac{KP}{KL} = \frac{3}{4}$.



20. Yes. If two triangles are congruent, then corresponding angles are congruent and corresponding sides are proportional with a ratio of $\frac{1}{1}$, so the triangles are similar.

21. Possible answer: You would measure the height and weight of the real ice cream cone and the height of the sculpture.

$$\frac{W_{\text{sculpture}}}{W_{\text{ice cream cone}}} = \left(\frac{H_{\text{sculpture}}}{H_{\text{ice cream cone}}}\right)^3$$

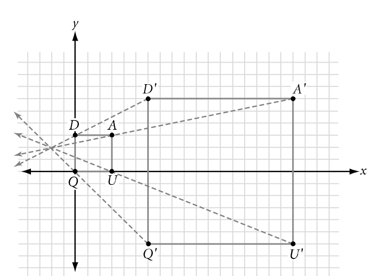
If you don't know the height of the sculpture, you could estimate it by taking a photo with a person standing in the window and setting up a ratio, for example,

$$\frac{H_{\text{person}}}{H_{\text{person's photo}}} = \frac{H_{\text{sculpture}}}{H_{\text{sculpture's photo}}}$$

22. Since they share $\angle M$ and both triangles have a right angle, the triangles are similar by the AA Similarity conjecture. From the proportion $\frac{h}{7} = \frac{24}{25}$, $h = 6.72$.

23. $\triangle LPR \sim \triangle TPU$ by AA; $x = 24$. Since $\overline{LR} \parallel \overline{UT}$, $\angle LRU \cong \angle RUT$ and $\angle LTU \cong \angle RLT$ by the Alternate Interior Angles Conjecture, so $\triangle LPR \sim \triangle TPU$ by AA. Therefore the sides are proportional, so $\frac{x}{32} = \frac{27}{36}$. Solving for x gives $x = 24$.

24. $(-2, 2)$; $(x, y) \rightarrow (4x + 6, 4y - 6)$



25. $x = 15$. By the Angle Bisector/Opposite Side Conjecture, $\frac{x}{10} = \frac{30}{20}$. Solving for x gives $x = 15$.

26. $x = 4$; The Proportional Parts conjecture tells us that the median is also proportional. Since the sides are all proportional. $\triangle ABD \sim \triangle EFH$ and $\triangle BDC \sim \triangle FHG$.

27. $w = 32$, $x = 24$, $y = 40$, $z = 126$. Apply the Extended Parallel/Proportionality Conjecture to find w, x, and y.

$$\frac{36}{24} = \frac{48}{w}$$
$$\frac{3}{2} = \frac{48}{w}$$
$48 = 16 \cdot 3$, so $w = 16 \cdot 2 = 32$.

$$\frac{36}{18} = \frac{48}{x}$$
$$\frac{2}{1} = \frac{48}{x}$$
$$x = 24$$

$$\frac{36}{30} = \frac{48}{y}$$
$$\frac{6}{5} = \frac{48}{y}$$
$48 = 8 \cdot 6$, so $y = 8 \cdot 5 = 40$.

The small triangle at the top of the figure (with base of length 42) is similar to the large triangle (with base of length z). Use a proportion based on this similarity to find z.

$$\frac{36 + 24 + 18 + 30}{z} = \frac{36}{42}$$
$$\frac{108}{z} = \frac{6}{7}$$
$108 = 18 \cdot 6$, so $z = 18 \cdot 7 = 126$.

TAKE ANOTHER LOOK

1. a. $(x, y) \rightarrow (2x + 3, 2y - 2)$

b. $(x, y) \rightarrow (2x + 3, -2y + 2)$

c. $(x, y) \rightarrow (-2x - 3, -2y + 2)$

2. Figure A shows the positions of the Moon and the Sun during a total eclipse, relative to an observer at point A. The Moon blocks all of the Sun except the Sun's corona from Earth's view. $\triangle ABC \sim \triangle ADE$ by the AA Similarity Conjecture. AB is the approximate distance from Earth to the Moon. BC is the approximate diameter of the Moon. AD is the approximate distance from Earth to the Sun. DE is the approximate diameter of the Sun. (*Note:* The figures on the next page are not drawn to scale. Because the actual distances are so great, relative to the diameters, don't worry that $\overline{AB}$ and $\overline{AD}$ are not precisely tangent to the spheres of the Moon and the Sun; nor will we worry about what points on the Moon and the Sun we use in measuring the distance from Earth.)

By similar triangles, $\frac{AB}{BC} = \frac{AD}{DE}$. If the Moon were smaller (or farther away), these ratios wouldn't be equal—the Moon wouldn't block all of the Sun's light (Figure B). If the Moon were larger (or closer), it would block an area larger than that of the Sun—we wouldn't be able to see the Sun's corona during an eclipse (Figure C).

The combination of the Moon's distance and diameter makes it fit "just right" in front of the Sun during a total eclipse.

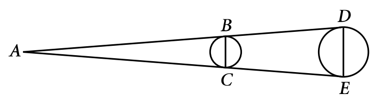


Figure A

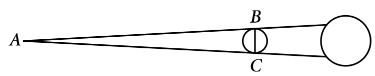


Figure B

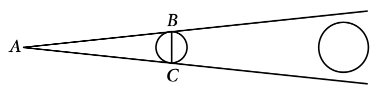


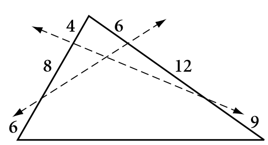
Figure C

3. Possible answers include any two triangles, one of whose sides have lengths a, b, and c, with the sides of the other having lengths b, c, and $\frac{c^2}{b}$. If one similar triangle has side lengths a, b, and c, then the other triangle has side lengths ar, br, and cr, where r is the scale factor. Because two sides must be congruent without the triangles being congruent, let $b = ar$ and $c = rb$. Then $c = ar \cdot r = ar^2$. To find values of r for which this relationship holds, use the Triangle Inequality Conjecture. $a + b > c$, $a + ar > ar^2$. (Similar equations that reduce to this occur for other pairs of sides.) Solve the inequality:

$$r < \frac{1 + \sqrt{5}}{2}$$

In other words, r must be less than the golden ratio.

4. The converse is not true. One counterexample:



5. Conjecture: If three sides of one triangle are parallel to the three sides of another triangle, then the triangles are similar.

Proof: Extend all sides so that they intersect. Then for each angle, apply the Corresponding Angles Conjecture twice.