Name KEY
Period ____

Precalculus/Trig
Section 5.3 Worksheet

Solve each of the following trigonometric equations. Write "no solution" if that is the case.
Check for extraneous solutions.

A) $2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$ ← reference $\angle$ for $\cos\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$

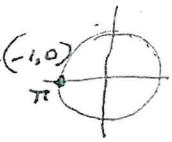
Cos is NEGATIVE in II & III

$\theta = \frac{5\pi}{6} + 2\pi n$, $\frac{7\pi}{6} + 2\pi n$

B) $5\cos\theta - 3 = 4\cos\theta - 4$

$-4\cos\theta + 3 \quad -4\cos\theta + 3$

$\cos\theta = -1$



$\theta = \pi + 2\pi n$

C) $2\cos^2\theta - 1 = 0$

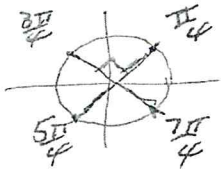
$2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{1}}{\sqrt{2}}$

$\cos\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$ ← All 4 QUADRANTS

ref $\angle$ is $\frac{\pi}{4}$



$\theta = \frac{\pi}{4} + \frac{\pi}{2}n$

D) $\sec\theta\sin\theta = -2\sin\theta$

SET = 0.

$\sec\theta\sin\theta + 2\sin\theta = 0$ ← FACTOR OUT GCF

$\sin\theta(\sec\theta + 2) = 0$

$\sin\theta = 0$ | $\sec\theta + 2 = 0$

$\theta = \pi n$ ← BOTH SOLUTIONS

$\sec = -2$ (recip.)

$\cos\theta = -\frac{1}{2}$ (recip.)

$\frac{\pi}{3}$ in II & III

$\theta = \frac{2\pi}{3} + 2\pi n$; $\frac{4\pi}{3} + 2\pi n$

E) $2\sec^2\theta + \tan^2\theta - 3 = 0$

PYTHAG IDENTITY

$2(1 + \tan^2\theta) + \tan^2\theta - 3 = 0$

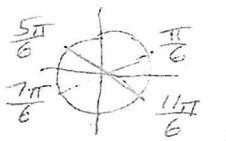
$2 + 2\tan^2\theta + \tan^2\theta - 3 = 0$

$3\tan^2\theta - 1 = 0$

$3\tan^2\theta = 1$

$\sqrt{\tan^2\theta} = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{1}}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}}$

$\tan\theta = \pm\frac{\sqrt{3}}{3}$ ← $\frac{\pi}{6}$ in ALL 4 QUADRANTS



$\theta = \frac{\pi}{6} + \pi n$ & $\frac{5\pi}{6} + \pi n$

F) $\csc^2\theta = 3\csc\theta + 4$

$\csc^2\theta - 3\csc\theta - 4 = 0$

$(\csc\theta - 4)(\csc\theta + 1) = 0$

$\csc\theta - 4 = 0$ | $\csc\theta + 1 = 0$

$\csc\theta = 4$ (recip) $\sin\theta = \frac{1}{4}$

$\csc\theta = -1$ (recip) $\sin\theta = -1$

$\theta = \sin^{-1}\left(\frac{1}{4}\right)$

$\theta = \frac{3\pi}{2} + 2\pi n$

$\theta = \sin^{-1}\left(\frac{1}{4}\right) + 2\pi n$

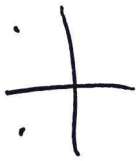
$\theta = \left[\pi - \sin^{-1}\left(\frac{1}{4}\right)\right] + 2\pi n$

BOTH SOLUTIONS

Turn over for more problems.

G) $\cos\frac{\theta}{2} = -\frac{\sqrt{2}}{2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$



$\frac{1}{2}X = \frac{3\pi}{4} + 2\pi n \quad \frac{1}{2}X = \frac{5\pi}{4} + 2\pi n$

$X = \frac{3\pi}{2} + 4\pi n \quad X = \frac{5\pi}{2} + 4\pi n$

H) $2\cos 3\theta = -\sqrt{3}$

$\cos 3\theta = \frac{-\sqrt{3}}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$3X = \frac{5\pi}{6} + 2\pi n \quad 3X = \frac{7\pi}{6} + 2\pi n$

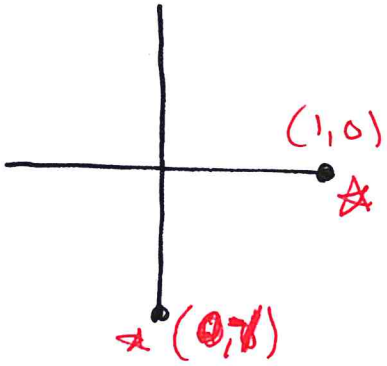
$X = \frac{5\pi}{18} + \frac{2}{3}\pi n$

$X = \frac{7\pi}{18} + \frac{2}{3}\pi n$

For these questions only give the values of θ in the interval $[0, 2\pi)$ Check for extraneous solutions and write "no solution" if that is the case.

I) $\sin\theta + 1 = \cos\theta$

↑ going around the unit circle, where can I add 1 to sine and have it equal cosine?



$\frac{3\pi}{2}, 0$

J) $4\sin\theta = -12$

$\sin\theta = -12/4 = -3$

not possible

(the range of sine is $[-1, 1]$)