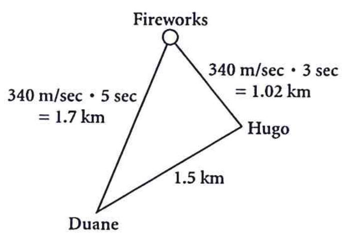
minute hand has already crossed the hour hand, so try 3:16. At 3:16, the hour hand has gone $\frac{16}{60}$ of the way from the 3 to the 4, while the minute hand has gone $\frac{1}{5}$ of the way from the 3 to the 4. Again, compare the fractions by changing them to decimals: $\frac{16}{60} = 0.2\overline{6}$ and $\frac{1}{5} = 0.2$. $\frac{16}{60} > \frac{1}{5}$, so at 3:16, the minute hand has not yet crossed the hour hand. Therefore, the hands overlap sometime between 3:16 and 3:17, or between 16 and 17 minutes after 3:00.

15. 120. n concurrent lines divide the plane into $2n$ parts; if $n = 60$, $2n = 120$. (See solution for Lesson 4.5, Exercise 24.)

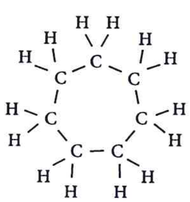
16. Any point at which the x-coordinate is either 1 or 7 and the y-coordinate does not equal 3, or the points (4, 6) or (4, 0).

17. 230°

18. Hugo and Duane can locate the site of the fireworks by creating a diagram using SSS.



19.



Because "hept" means 7, cycloheptane has 7 C's. There are 2 H's branching off each C, so there are 14 H's. In general, a cycloparaffin has n C's and $2n$ H's, so the general rule for a cycloparaffin is C_nH_{2n}.

Performance Task

If Michaels remembers which rope he tied to which initial point, he would be able to locate the treasure. The triangle congruence is SSS because he knows the lengths of the two ropes and the distance between Skull Rock and Hangman's tree stays the same. Using a compass setting for 55 m and 105m, the only intersection on the island would be Freebooter Cemetery for the buried treasure.

Extension

If any two segments coincide, all three will. (This happens when the three segments are drawn from the vertex angle of an isosceles triangle or from any vertex of an equilateral triangle.)

CHAPTER 4 REVIEW

Exercises

1. **a.** true, **b.** true, **c.** false, **d.** false, **e.** both

2. The Triangle Sum Conjecture states that the sum of the measures of the angles in every triangle is 180°. Possible answers: It applies to all triangles; many other conjectures rely on it.

3. Possible answer: The angle bisector of the vertex angle is also the median and the altitude.

4. The distance between A and B is along the segment connecting them. The distance from A to C to B can't be shorter than the distance from A to B. Therefore, $AC + CB > AB$. Points A, B, and C form a triangle. Therefore, the sum of the lengths of any two sides is greater than the length of the third side.

5. SSS, SAS, ASA, or SAA

6. In some cases, two different triangles can be constructed using the same two sides and non-included angle.

7. 1

8. Annie is correct. Given $\triangle NOT$. $\angle N + \angle O + \angle T = 180°$ by the Triangle Sum Conjecture. So $2y + 2x + 2z = 180°$, or $y + x + z = 90°$. Subtracting z from each side gives $y + x = 90° - z$. If the bisectors met at right angles, then $y + x = 90°$ in $\triangle TPN$.

9. Cannot be determined. SSA does not guarantee congruence.

10. $\triangle TOP \cong \triangle ZAP$ by SAA.

11. $\triangle MSE \cong \triangle OSU$ by SSS.

12. Cannot be determined. SSA does not guarantee congruence.

13. $\triangle TRP \cong \triangle APR$ by SAS.

14. $\triangle CGH \cong \triangle NGI$ by SAS. Use the Converse of the Isosceles Triangle Conjecture to get $\overline{HG} \cong \overline{IG}$, and use the vertical angles.

15. Cannot be determined. You have two pairs of congruent sides, but there is no information about the third pair of sides or about any of the pairs of angles.

16. $\triangle ABE \cong \triangle DCE$ by SAA or ASA.

17. $\triangle ACN \cong \triangle RBO \cong \triangle OBR$ by SAS. In a regular polygon, all sides are congruent and all interior angles are congruent.

18. $\triangle AMD \cong \triangle UMT$ by SAS; $\overline{AD} \cong \overline{UT}$ by CPCTC.

19. Cannot be determined. AAA does not guarantee congruence.

20. Cannot be determined. SSA does not guarantee congruence.

21. $\triangle TRI \cong \triangle ALS$ by SAA; $\overline{TR} \cong \overline{AL}$ by CPCTC. Use alternate interior angles to show the triangles are congruent.

22. $\triangle SVE \cong \triangle NIK$ by SSS; $\overline{EI} \cong \overline{KV}$ by the overlapping segments property. $EV + VI = EI$ and $KI + IV = KV$. Because $EV = KI$ and $VI = IV$, $EI = KV$ and thus $\overline{EI} \cong \overline{KV}$.

23. Cannot be determined. Parts don't correspond. Notice that the shared side is not opposite congruent angles.

24. Cannot be determined. There is not sufficient information to determine that the triangles are congruent. By the AIA Conjecture, $\angle MNT \cong \angle CTN$. With the shared side, this gives SSA, which does not guarantee that the triangles are congruent. You are not able to determine whether $\overline{MT} \parallel \overline{NC}$, so you cannot tell whether quadrilateral $NCTM$ has one pair of opposite parallel sides or two pairs, and thus cannot determine whether it is a parallelogram or a trapezoid.

25. $\triangle LAZ \cong \triangle IAR$ by ASA, $\triangle LRI \cong \triangle IZL$ by ASA, and $\triangle LRD \cong \triangle IZD$ by ASA. There are actually two isosceles triangles in the figure, $\triangle LAI$ and $\triangle LDI$, and there are three pairs of congruent triangles. For $\triangle LAZ \cong \triangle IAR$, use the shared angle, $\angle A$. For $\triangle LRI \cong \triangle IZL$, use the common side, $\overline{LI}$. Also, $m\angle RLI = m\angle ZIL$ (Isosceles Triangle Conjecture), and $m\angle RLZ = m\angle ZIR$ (given), so by subtraction, $m\angle ZLI = m\angle RIL$, and thus, $\angle ZLI \cong \angle RIL$. Thus, the triangles are congruent by ASA. Because $\angle ZLI \cong \angle RIL$, $\overline{DI} \cong \overline{DL}$ by the Converse of the Isosceles Triangle Conjecture. Using this pair of sides, the marked angles, and the vertical angles, $\triangle LRD \cong \triangle IZD$ by ASA.

26. $\triangle PTS \cong \triangle TPO$ by ASA or SAA; yes. Use the shared side to show that the triangles are congruent. By the Converse of the Parallel Lines Conjecture, you can show that $\overline{PS} \parallel \overline{TO}$ and $\overline{PO} \parallel \overline{ST}$, so $STOP$ is a parallelogram.

27. $\triangle ANG$ is isosceles, so $\angle A \cong \angle G$. Then, $m\angle A + m\angle N + m\angle G = 188°$. The sum of the measures of the three angles of a triangle must be 180°, so an isosceles triangle with the given angle measures is impossible.

28. $\triangle ROW \cong \triangle NOG$ by ASA, implying that $\overline{OW} \cong \overline{OG}$. However, the two segments shown are not the same length.

29. c is the longest segment, and a and g are the shortest. Apply the Side-Angle Inequality Conjecture to all three triangles in succession. First, in the triangle with side lengths a and g, the third angle measure is 30°, so this triangle is isosceles with $a = g < f$. Now look at the triangle with side lengths d, e, and f. The angle opposite the side with length e measures 60°, because the measure of this angle + 30° + 90° = 180°. So the angle opposite the side with length f must also measure 60°. Thus, this triangle is equilateral, and $f = d = e$. Finally, look at the right triangle. The unmarked angle measures 45°, so this is an isosceles right triangle with $b = d$. In this triangle, c (the hypotenuse) is the longest side, so $b = d < c$. Putting the inequalities from the three triangles together, you have $a = g < f = d = e = b < c$, so c is the longest segment and a and g are the shortest.

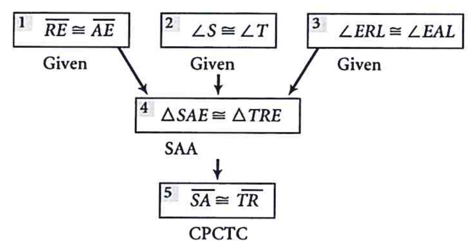
30. $x = 20°$. Apply the Triangle Sum Conjecture to both the large triangle and the triangle containing the angle marked as 100°. From the large triangle, $2a + 2b + x = 180°$, and from the triangle with the 100° angle, $a + b + 100° = 180$, or $a + b = 80°$, which is equivalent to $2a + 2b = 160°$. Substituting 160° for $2a + 2b$ in the equation $2a + 2b + x = 180°$ gives $x = 20°$.

31. Yes

Given: $\overline{RE} \cong \overline{AE}$, $\angle S \cong \angle T$, $\angle ERL \cong \angle EAL$

Show: $\overline{SA} \cong \overline{TR}$

Flowchart Proof



32. Yes

Given: $\angle A \cong \angle M$, $\overline{AF} \perp \overline{FR}$, $\overline{MR} \perp \overline{FR}$

Show: $\triangle FRD$ is isosceles

Flowchart Proof