Today - the exact same math extended to all 4 Quadrants.

Unit Circle ($r=1$)

reciprocals

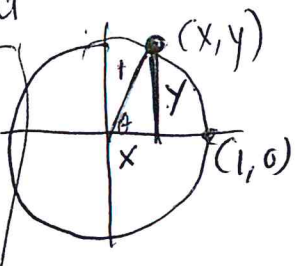
$\sin\theta = \frac{y}{1}$ = y-coordinate $\quad \cos\theta = x$-coord

$\csc\theta = \frac{1}{y} \quad \sec\theta = \frac{1}{x}$

Quotient

$\tan = \frac{\sin}{\cos} = \frac{y}{x}$

$\cot = \frac{\cos}{\sin} = \frac{x}{y}$



Any Circle

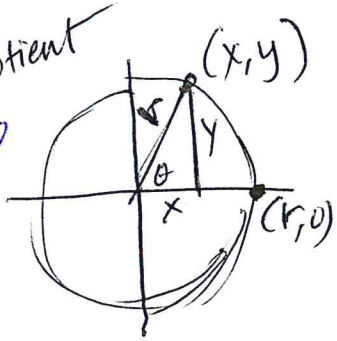
recip.

$\sin\theta = \frac{y}{r}$ = y Coordinate $\quad \cos\theta = \frac{x}{r}$ x coord

$\csc\theta = \frac{r}{y}$ $y \neq 0 \quad \sec\theta = \frac{r}{x}$ $x \neq 0$

$\tan = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x}$ quotient $x \neq 0$

$\cot = \frac{x}{y}$ $y \neq 0$



II +,+ Sin +,+ I

+	+
neg	neg

III −,− IV +,−

II COS I

neg	+
neg	+

x

III IV

II tangent I

neg	+ $\frac{\sin\theta}{\cos\theta}$
+	neg

III IV

~~csc~~ cosecant $\frac{r}{y}$

+	+
neg	neg

secant (like cos) $\frac{r}{x}$

neg	+
neg	+

II

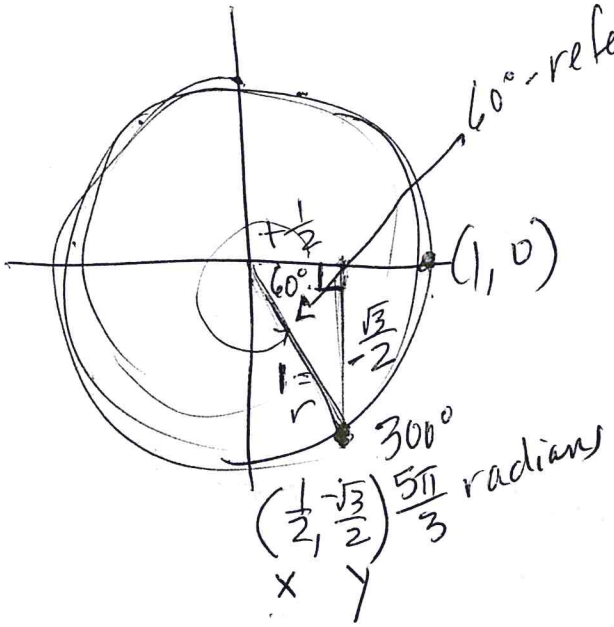
cotangent I

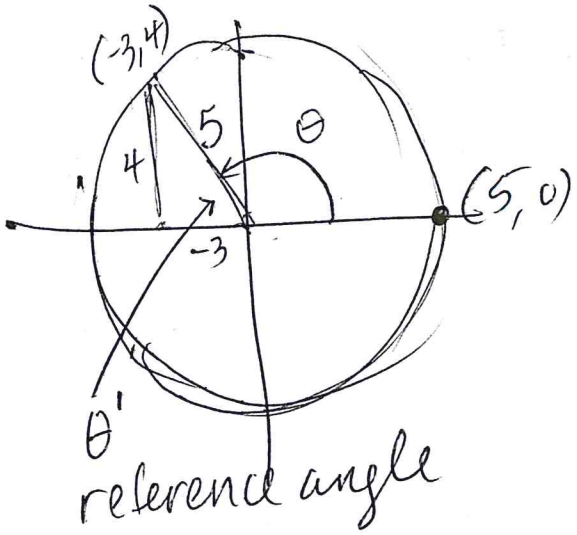
neg	pos
pos	neg

III IV

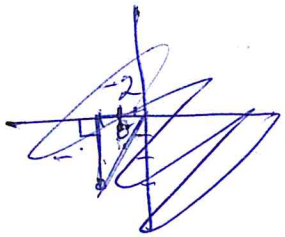
r is always positive (distance, hypotenuse), x & y are coordinates, so + or − depends on Quadrant.

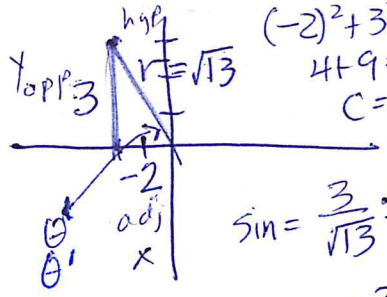
Reference angle - θ' is the angle between the rotated ray and the x-axis. The one that would make a right $\triangle$ to the x-axis





Ex) Like #1 Let (-2, 3) be a pt on a circle. terminal side of θ angle. Find sin, cos, tan.





$(-2)^2 + 3^2 = c^2$

$4 + 9 = c^2$

$c = \sqrt{13}$

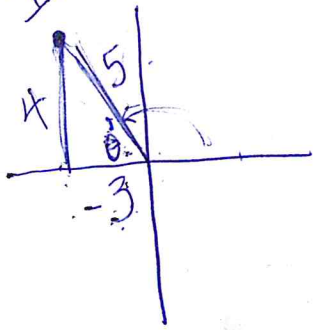
$\sin = \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\cos = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

$\tan = \frac{3}{-2}$

Ex) 2 $\sin\theta = \frac{4}{5}$ $\tan\theta < 0$ find $\cos\theta$ & $\tan\theta$

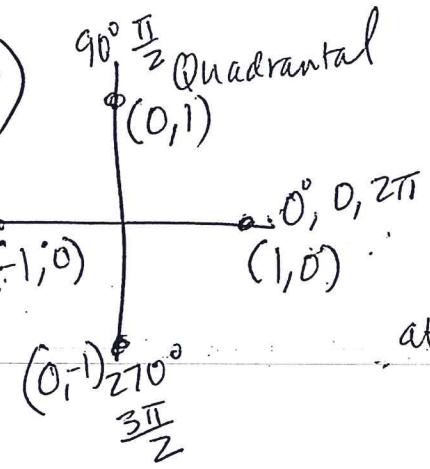
II



$\cos\theta = \frac{-3}{5}$

$\tan\theta = \frac{4}{-3} = -\frac{4}{3}$

EX 3)



cot = cos/sin = 1/0 undef

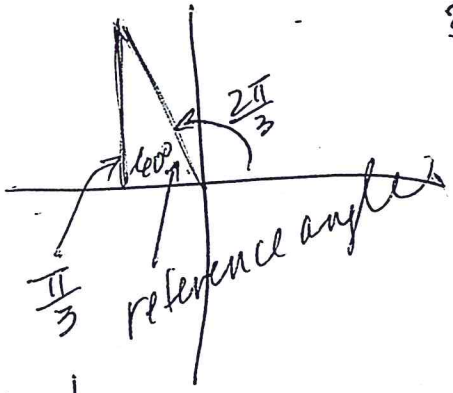
2π, 0, π y-coord = 0

csc = 1/y = 1/0 undef

at π/2, 3π/2 x-coord = 0

sec θ, tan θ undefined

1/x = 0 y/x = 0

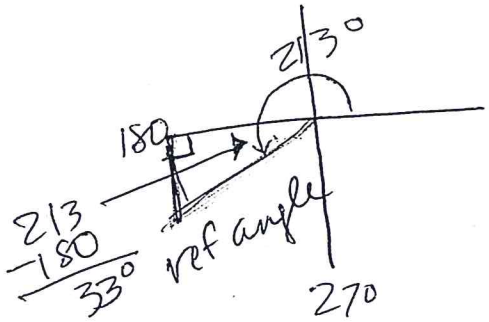


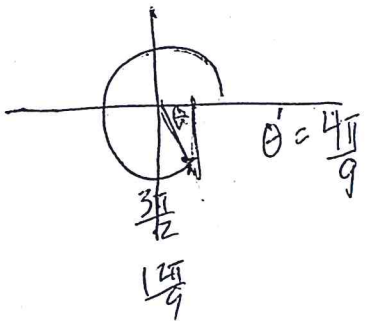
2π/3 rad.

Find the reference angle for

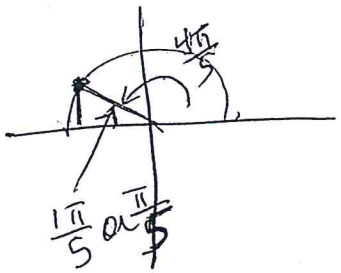
a) θ = 213° b) 14π/9

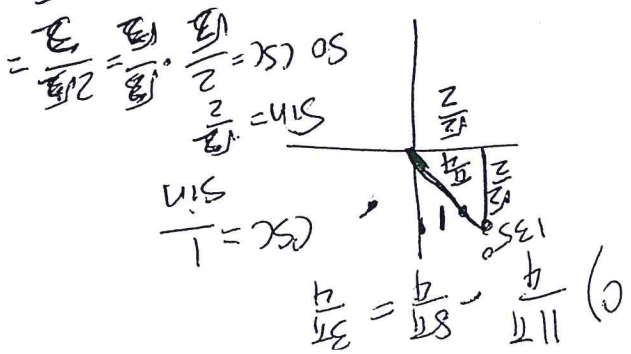
EX) 4

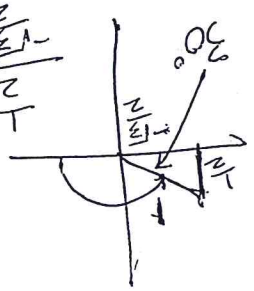


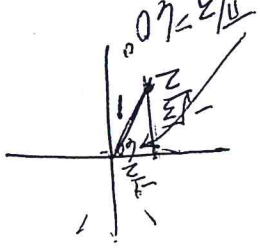


c) 4π/5

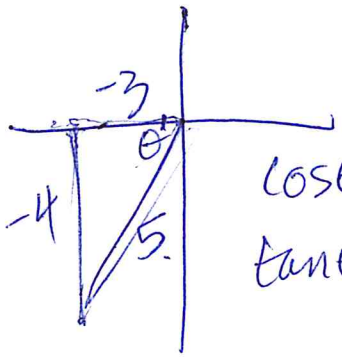








Quad II $\sin = \frac{-4}{5}$ EX 6)

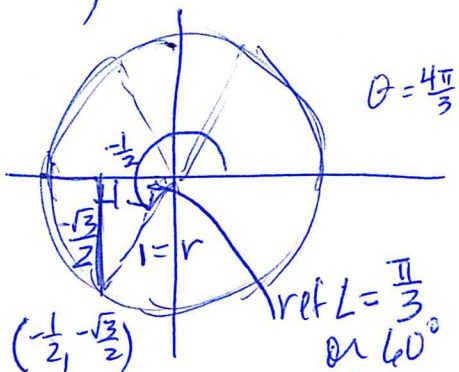


$\cos\theta = ? = \frac{-3}{5}$

$\tan\theta = ? \frac{-4}{-3} = \frac{4}{3}$

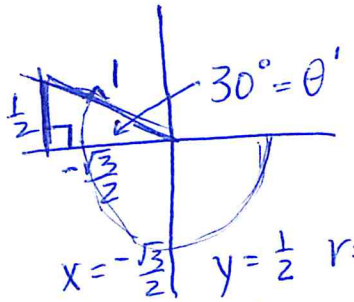
EX 5) Using Reference Angles

a) $\cos \frac{4\pi}{3}$



$\cos = \frac{x}{r} = \boxed{\frac{-1}{2}}$

b) $\tan(-210)$

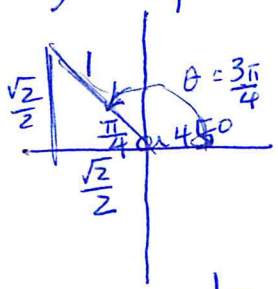


$x = -\frac{\sqrt{3}}{2}$ $y = \frac{1}{2}$ $r = 1$

$\tan = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{1}{-\sqrt{3}}$

$= \boxed{\frac{-\sqrt{3}}{3}}$

c) csc of $\frac{11\pi}{4}$ $-\frac{8\pi}{4} = \frac{3\pi}{4}$



$\csc = \frac{1}{\sin}$

$\sin = \frac{y}{r}$

$\csc = \frac{r}{y}$

$= \frac{1}{\frac{\sqrt{2}}{2}} = \boxed{\sqrt{2}}$