Review for Fall Final Exam Precalculus Name: ____________________

Calculators are permitted on the Fall Final Exam. You may make a 3 inch by 5 inch notecard (using the front and back) for the final exam. Fall Final Exam questions were taken from the quizzes from chapters 4 and 5 and the following problems:

1) Find the point (x, y) on the unit circle that corresponds to the central angle $\frac{5\pi}{3}$.

$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

2) Evaluate the six trig functions of θ if $\theta = -\frac{5\pi}{4}$.

$\sin\theta = \frac{\sqrt{2}}{2}$ $\tan\theta = -1$ $\csc\theta = \sqrt{2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$ $\cot\theta = -1$ $\sec\theta = -\sqrt{2}$

3) Evaluate the six trig functions of θ if $\theta = \frac{3\pi}{2}$.

$(0, -1)$ cos θ sin; $\tan\theta = \frac{-1}{0}$ undef; $\cot\theta = 0$; $\cos = 0$; $\sec\theta =$ undef.; $\csc\theta = -1$

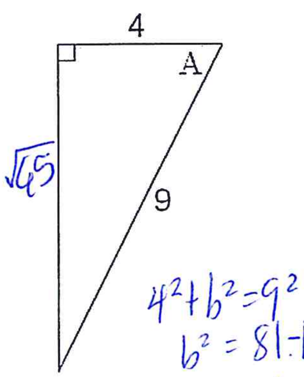
4) Use a calculator to evaluate to thousandths place:

A) $arc\sec(4.1)$ set to radians; $2^{nd}\cos(1/4.1)$ 1.324

B) $\csc(160°)$ set to degrees; $1/\sin 160°$ 2.924

C) $\cot\left(\frac{3\pi}{20}\right)$ $1/\tan(3\pi/20)$ set to radians 1.963

5) Find the exact values of the six trig functions of the angle A shown in the figure.



$4^2 + b^2 = 9^2$
$b^2 = 81 - 16$
$b^2 = 65$ $b = \sqrt{65}$

sin A= $\frac{\sqrt{65}}{9}$ csc A= $\frac{9}{\sqrt{65}} = \frac{9\sqrt{65}}{65}$

cos A= $\frac{4}{9}$ sec A= $\frac{9}{4}$

tan A= $\frac{\sqrt{65}}{4}$ cot A= $\frac{4}{\sqrt{65}} = \frac{4\sqrt{65}}{65}$

6) Use the vertical format & trig identities to transform the left side of the equation into the right side.

A) $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \sec\theta\csc\theta$

$\frac{\sin^2\theta}{\cos\theta\sin\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta}$

$\frac{1}{\cos\theta\sin\theta}$

$\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$\sec\theta\csc\theta = \sec\theta\csc\theta$ ✓

B) $\frac{\cot\theta + \tan\theta}{\cot\theta} = \sec^2\theta$

$\frac{\cot\theta}{\cot\theta} + \frac{\tan\theta}{\cot\theta}$

$1 + \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$

$1 + \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta$ ✓

$\sec^2\theta = \sec^2\theta$

7) The point $(-2,5)$ is on the terminal side of an angle θ in standard position. Determine the exact values of the six trig functions of θ.

$\sqrt{29}$

$2^2+5^2=29$

$c=\sqrt{29}$

$\sin\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$ $\csc\theta = \frac{\sqrt{29}}{5}$ $\tan\theta = \frac{-5}{2}$

$\cos\theta = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$ $\sec\theta = \frac{-\sqrt{29}}{2}$ $\cot\theta = \frac{-2}{5}$

8) Find the value of $\tan\theta$ given $\sin\theta = \frac{1}{5}$ and $\cos\theta < 0$

$a^2+1^2=5^2$

$a=\sqrt{24}=-2\sqrt{6}$

$-\frac{1}{2\sqrt{6}} = \frac{-\sqrt{6}}{12}$

9) Find the reference angles: A) $262°$ B) $\frac{7\pi}{9}$

A) $82°$

B) $\frac{2\pi}{9}$

10) If $\sec\theta = \sqrt{10}$ in Quadrant IV, find $\tan\theta$.

$\cos = \frac{1}{\sqrt{10}}$ $\tan = -\frac{3}{1} = -3$

11) Convert $1\frac{4}{5}$ revolutions counterclockwise from positive x-axis to radians.

$2\pi + \frac{4}{5}\cdot 2\pi$ $2\pi + \frac{8}{5}\pi = \frac{10\pi}{5} + \frac{8\pi}{5} = \frac{18\pi}{5}$ radians

12) A 15-foot ladder makes an angle of $25°$ with the ground as it leans against a house. Approximately how far up the house does the ladder reach?

round to thousandths

$15\cdot\sin 25° = \frac{x}{15}\cdot 15$

$x \approx 6.339$ ft

13) Find the quadrant in which θ lies if $\sec\theta < 0$ and $\sin\theta > 0$.

II

14) Find $\sec\frac{\pi}{2}$

$(0,1)$ $\sec = \frac{1}{\cos}$

undef

$1/0 =$ undef.

15) Find the value of $\sin\theta$ when $\sec\theta = -3$ and $\frac{\pi}{2} < \theta < \pi$.

$1^2+b^2=3^2$

$b^2=8$

$b=2\sqrt{2}$

$\frac{1}{\cos} = -3$

$\cos = \frac{-1}{3}$

$\frac{2\sqrt{2}}{3}$

16) Find the amplitude, period, phase shift, and vertical shift of $y = -3\cos\left(2x - \frac{\pi}{3}\right) - 1$

$\frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

amplitude $= 3$ ~~period~~ vertical shift $= -1$ (down 1) phase shift $=$ right $\frac{\pi}{6}$ period $= \pi$

17) Find the trig function with period $= 2\pi$, domain: all $x \neq n\pi$, and range: $(-\infty,-1]$ $[1,\infty)$.

$y = \csc x$

18) Find the trig function with period $= \pi$, domain: all $x \neq n\pi$, and range: $(-\infty,\infty)$.

$y = \cot x$

tan or cot

19) Name the trig functions that are even and symmetric to the y-axis.

cosine, secant all others are odd

20) Find the exact value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

$\frac{-\pi}{3}$

range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

21) Find the exact value of $\arctan\left(-\sqrt{3}\right)$.

$\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)$ $\quad -\frac{\pi}{3}$

22) Find the exact value of $\sec\left[\arctan\left(-\frac{5}{12}\right)\right]$.

$\tan^{-1}\left(\frac{-5}{12}\right)$ $\quad \sec\theta = \frac{13}{12}$

23) Write a simplified algebraic expression that is equivalent to $\cot(\arccos 3x)$.

$\frac{3x}{\sqrt{1-9x^2}}$ $\quad 9x^2 + b^2 = 1 \quad b = \sqrt{1-9x^2}$

24) An engineer erects a 100-foot vertical cellular-phone tower. Find the angle of elevation in degrees to the top of the tower from a point on level ground 90 feet from its base.

$\tan x = \frac{100}{90}$ $\quad \tan^{-1}\left(\frac{100}{90}\right) =$

25) **True** or **False**: $\csc(-\theta) = \dfrac{\cot(-\theta)}{\sec\theta}$.

False

$\csc(-\theta) = \frac{1}{-\sin\theta}$ $\quad \cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{-\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{1} = \frac{-\cos^2\theta}{\sin\theta}$ False

26) Let $\sin\theta = \dfrac{4}{7}$ and $\tan\theta < 0$. What is the value of $\sec\theta$?

$49 - 16 = 33$ $\quad \frac{7}{-\sqrt{33}} = \frac{-7\sqrt{33}}{33}$

27) Simplify $\tan\theta - \tan\theta\sin^2\theta$ by factoring out the GCF then using fundamental identities.

$\tan\theta(1-\sin^2\theta)$ $\quad \tan\theta\cdot\cos^2\theta = \frac{\sin\theta}{\cos\theta}\cos^2\theta = \sin\theta\cos\theta$

28) Use trigonometric substitution to write $\sqrt{9x^2+81}$ as a trigonometric function of θ, where $0 < \theta < \dfrac{\pi}{2}$.

Use $x = 3\cot\theta$.

$\sqrt{9(3\cot\theta)^2+81}$ $\quad \sqrt{81\cot^2\theta+81}$ $\quad \sqrt{81(\cot^2\theta+1)}$ $\quad \sqrt{81\csc^2\theta} = 9\csc\theta$ $\quad \csc^2\theta = \cot^2\theta + 1$

29) Solve $\cos^2\theta = \cos\theta$ over the set of real numbers where n ={integers}.

$\cos^2\theta - \cos\theta = 0$ $\quad \cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0$: $\frac{\pi}{2} + \pi n$ $\quad \cos\theta = 1$: $2\pi n$

30) Solve $\sin^2\theta - 3\sin\theta + 2 = 0$ for the interval $[0, 2\pi)$.

$(\sin\theta - 2)(\sin\theta - 1) = 0$

$\sin\theta = 2$ not in domain $\quad \sin\theta = 1$ $\quad \frac{\pi}{2}$

31) Solve $2\sin^2 x + 3\cos x - 3 = 0$ for the interval $[0, 2\pi)$.

$c^2 + s^2 = 1 \quad \sin^2 = 1 - c^2$

$2(1-\cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$-2\cos^2 x + 3\cos x - 1 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$: $\frac{\pi}{3}, \frac{5\pi}{3}$ $\quad \cos x = 1$: 0

$0, \frac{\pi}{3}, \frac{5\pi}{3}$

32) Write $\csc(\arccos x)$ as an algebraic expression.

$\csc = \frac{hyp}{opp}$

1, $\sqrt{1-x^2}$, θ, x

$\frac{1}{\sqrt{1-x^2}}$

33) Find the exact value of $\sin\left(\arccos\frac{-1}{2}\right)$.

$\cos^{-1}\left(\frac{-1}{2}\right)$

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\frac{2\pi}{3}$

34). **Let** $\cos\theta = -\frac{4}{5}$ and $\sin(\theta) > 0$ **then find** $\tan\theta$

5, 3, −4

$\frac{-3}{4} = \tan\theta$

35). **Let** $\cos\theta = -\frac{5}{13}$ and $\tan(\theta) > 0$ **then find** $\sin(\theta)$.

−5, −12, 13

$\frac{-12}{13} = \sin\theta$

36). **Let** $\tan\theta = -\frac{8}{15}$ and $\sin(\theta) < 0$ **then find** $\cos(\theta)$.

15, −8, 17

$\frac{15}{17}$

37). **A ship is 40 miles west and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?**

40, 90, port, 30, 53.13

$\tan^{-1}\left(\frac{3}{4}\right) = 36.870°$ ~~+90~~

~~N 126.870° W~~

N 53.130° E

38). **Use a general formula to represent the infinite solutions to** $2\cos^2\theta - \cos\theta = 1$. **Use n={integer}**

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ $\cos\theta = 1$

$\frac{-2}{2}$ 1

$2\pi/3$, $4\pi/3$

$\frac{2\pi}{3} + 2\pi n$

$\frac{4\pi}{3} + 2\pi n$

$2\pi n$ and