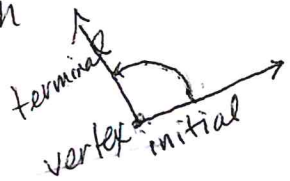
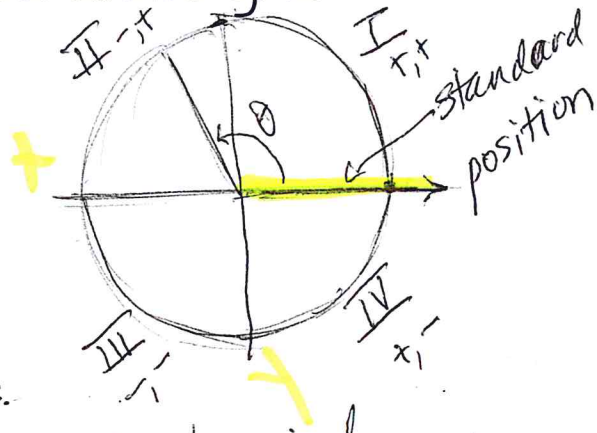
Precalculus Notes 4.1 - Radian and degree measure

- What is angle measure? how much one side of an angle rotates in order to end at the other side.



- Standard position for angles on coordinate grid with vertex at the origin:



  - Also called the initial side.
  - So the other side of the angle is the terminal side.
- Positive angles rotate counterclockwise
- Negative angles rotate clockwise

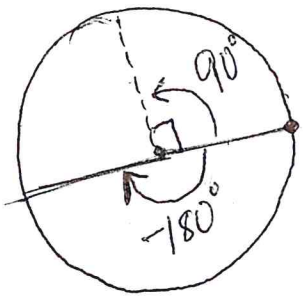
An angle can be formed from a rotation greater than 360°. Just keep rotating.

- Degrees vs. radians

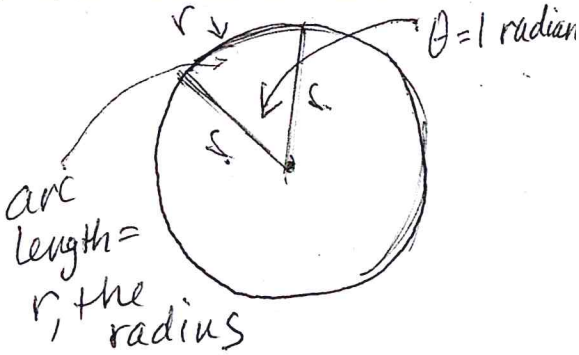
Degrees are familiar. One full circle is 360°

So 1/4 of a circle is 90°

1/2 of a circle is 180°



- Radians (demonstrated on the floor) - another way to measure angles! One radian is the measure of the central angle formed when the arc length between the sides is one radius of the circle. ~~There~~



So the whole circle contains 6+ radians (2π radians), making half of a circle be π radians, 1/4 of a circle is $\frac{\pi}{2}$ radians, etc.

To convert from degrees to radians use the fact that $\pi = 180°$

To find radians for a 60° angle, $\frac{60}{180} = \frac{x}{\pi}$ or radians = $\pi \cdot \frac{\text{degrees}}{180}$

Ex) $= \pi \cdot \frac{60}{180} = \frac{1}{3}\pi = \boxed{\frac{\pi}{3}}$

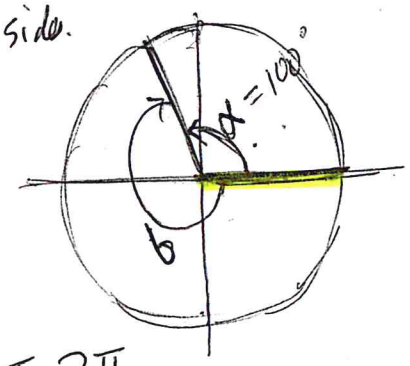
75° to radians $\frac{\pi \cdot 75}{180} = \frac{15}{36}\pi$ $\boxed{\frac{5}{12}\pi}$

Conversely, degrees = $180 \cdot \frac{\text{radians}}{\pi}$

Ex) Convert $\frac{5\pi}{6}$ to degrees $180 \cdot \frac{\frac{5\pi}{6}}{\pi}$ $\quad 180 \cdot \frac{5}{6} = 150°$

$\frac{5\pi}{4}$ to degrees $180 \cdot \frac{\frac{5\pi}{4}}{\pi} = \frac{180 \cdot 5}{4} = 45 \cdot 5 = 225°$

- Coterminal angles have the same ~~this~~ initial and terminal side.



How is that possible? A) opposite direction

$\frac{100}{180}\pi = \frac{5}{9}\pi$

angle − 360°

radians − 2π

$\frac{5}{9}\pi - 2\pi$

$\frac{5}{9}\pi - \frac{18}{9}\pi = \frac{-13\pi}{9}$

B) OR add 360° (or multiples)

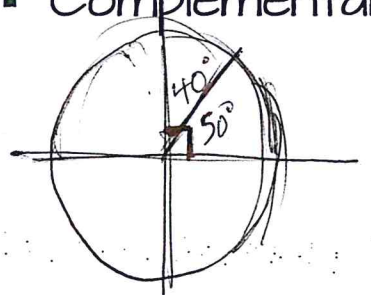
$100° + 360° = 460°$ ← coterminal

$\frac{5\pi}{9} + 2\pi$ $\quad \frac{18}{9} + \frac{5}{9} = \frac{23}{9}\pi$

Or add 2π

Must be able to do with degrees OR radians

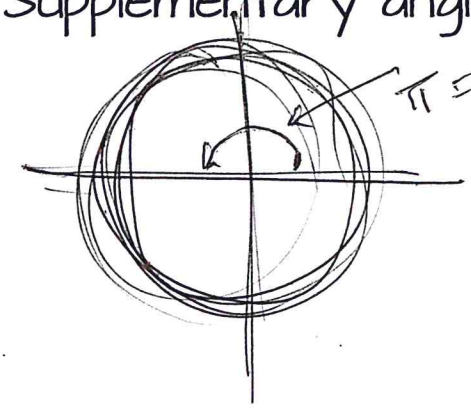
- Complementary angles add up to 90° degrees OR $\frac{\pi}{2}$ Radians



Complement of 50° =
$90-50=40^\circ$

Complement of $\frac{\pi}{3} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6}$

- Supplementary angles add up to 180° degrees OR 1π Radians



Supplement of 225°
$180-225=-45^\circ$
-45°

Supplement of $\frac{2\pi}{3} = \pi - \frac{2\pi}{3} = \frac{1}{3}\pi = \frac{\pi}{3}$

$\frac{3}{3}\pi - \frac{2}{3}\pi = \frac{1}{3}\pi = \frac{\pi}{3}$

Supplement of $\frac{7\pi}{4}$

$\pi - \frac{7\pi}{4} = \frac{4\pi}{4} - \frac{7\pi}{4} = \frac{-3\pi}{4}$

How to convert radians to degrees if radians are not in terms of π.

Ex) 5 radians to degrees

radians = $\pi \cdot \frac{\text{degrees}}{180}$

or degrees = $\frac{\text{radians}}{\pi} \cdot 180$

degrees = $\frac{5}{\pi} \cdot 180$

use calculator: (5/π) * 180
≈ 286°
or 286.5°

Ex) What quadrant does 5 radians lie in?

quadrant IV