CHAPTER 6 REVIEW

EXERCISES

1. Possible answer: The Inscribed Angle Conjecture is very important because several other conjectures build on it, and it can be used in many different situations.

2. Possible answers:

   With compass and straightedge: Draw two nonparallel chords, and construct their perpendicular bisectors. The intersection of their perpendicular bisectors is the center of the circle.

   With patty paper: Fold the paper along a diameter so that two semicircles coincide. Repeat with a different diameter. The center is the intersection of the two folds.

   With the right-angled corner of a carpenter's square: Place the corner in the circle so that it is an inscribed right angle. Trace the sides of the corner. Use the square to construct the hypotenuse of the right triangle (which is the diameter of the circle). Repeat. The center is the intersection of the two diameters.

3. The velocity vector is always perpendicular to the radius at the point of tangency to the object's circular path.

4. Sample answer: An arc measure is between 0° and 360°. An arc length is proportional to arc measure and depends on the radius of the circle.

5. 55°. Draw the radius to the point of tangency. By the Tangent Conjecture, the radius is perpendicular to the tangent, so a right triangle has been formed. Therefore the measure of the central angle that intercepts the arc of measure b is $90° - 35° = 55°$, so $b = 55°$ by the definition of the measure of an arc.

6. 65°. The 110° angle is an inscribed angle that intercepts an arc of measure $a + 155°$, so by the Inscribed Angle Conjecture, $110° = \frac{1}{2}(a + 155°)$; $220° = a + 155°$, and $a = 65°$.

7. 128°. Congruent chords intercept congruent arcs (Chord Arcs Conjecture), so the unmarked arc has measure c. Then, $2c + 104° = 360°$, so $2c = 256°$, and $c = 128°$.

8. 118°. First find the measure of either of the two vertical angles that form a linear pair with the angle of measure e. The measure of an angle formed by two intersecting chords is half the sum of the measures of the intercepted arcs (see Lesson 6.5, Exercises 16 and 17), so the measure of either one of these angles is $\frac{1}{2}(60° + 64°) = 62°$. Then, $e = 180° - 62°$ (Linear Pair Conjecture), so $e = 118°$.

9. 91°. The angle marked as a right angle intercepts a semicircle and is therefore inscribed in the opposite semicircle, so $d + 89° = 180°$, and $d = 91°$. (You could also use the Cyclic Quadrilateral Conjecture to see that the angle opposite the marked right angle is the supplement of a right angle, and therefore it is also a right angle. Then the intercepted arc of this right angle must measure 180°, so $d + 89° = 180°$.)

10. 66°. Look at either of the angles that form a linear pair with the 88° angle. The supplement of an 88° angle is a 92° angle. Because the measure of an angle formed by two intersecting chords is one-half the sum of the measures of their intercepted arcs (see Exercise 8), $92° = \frac{1}{2}(f + 118°)$, so $184° = f + 118°$, and $f = 66°$.

11. 125.7 cm. $C = 2\pi r = 2\pi(20) \approx 125.7$ cm.

12. 42.0 cm. $C = \pi d$, so $132 = \pi d$, and $d = \frac{132}{\pi} \approx 42.0$ cm.

13. 15π cm. $m\overset{\frown}{AB} = 100°$ (Chord Arcs Conjecture), so by the Arc Length Conjecture, the length of $\overset{\frown}{AB}$ is $\frac{100°}{360°}C = \frac{5}{18}(2\pi \cdot 27) = 15\pi$ cm.

14. 14π ft. In Lesson 6.6, Exercises 9 and 10, you discovered and proved a conjecture that says, "The measure of an angle formed by two intersecting secants through a circle is equal to one-half the difference of the larger arc measure and the smaller arc measure." First apply this conjecture to find $\overset{\frown}{DL}$: $50° = \frac{1}{2}(m\overset{\frown}{DL} - 60°)$, so $100° = m\overset{\frown}{DL} - 60°$, and $m\overset{\frown}{DL} = 160°$. By the Chord Arcs Conjecture, $m\overset{\frown}{CD} = m\overset{\frown}{OL}$, so $2m\overset{\frown}{CD} + 160° + 60° = 360°$. Then $2m\overset{\frown}{CD} = 140°$, and $m\overset{\frown}{CD} = 70°$. Now apply the Arc Length Conjecture. The length of $\overset{\frown}{CD}$ is $\frac{70°}{360°}C = \frac{7}{36}(2\pi \cdot 36 \text{ ft}) = 14\pi$ ft.

15. Look at the inscribed angles with measures 57° and 35°. By the Inscribed Angle Conjecture, the sum of the measures of their intercepted arcs is $2(57°) + 2(35°) = 184°$. However, the sum of these two arcs is a semicircle, and the measure of a semicircle is 180°, so this is impossible.

    Another way to look at this is to add the angle measures in the triangle. The third angle of the triangle must be a right angle because it is inscribed in a semicircle (Angles Inscribed in a Semicircle Conjecture). Therefore the sum of the three angles of the triangle would be $57° + 35° + 90° = 182°$, which is impossible by the Triangle Sum Conjecture.

16. By the Parallel Lines Intercepted Arcs Conjecture, the unmarked arc measures 56°. Then the sum of the arcs would be $84° + 56° + 56° + 158° = 354°$, but this is impossible because the sum of the measures of the arcs of a circle must be 360°.

17. $m\angle EKL = \frac{1}{2}m\widehat{EL} = \frac{1}{2}(180° - 108°) = 36° = m\angle KLY$. Therefore $\overline{KE} \parallel \overline{YL}$ by the Converse of the Parallel Lines Conjecture.

18. $m\widehat{JI} = 360° - 56° - 152° = 152° = m\widehat{MI}$. Therefore $m\angle J = m\angle M$ (Inscribed Angles Intercepting Arcs Conjecture), and $\triangle JIM$ is isosceles by the Converse of the Isosceles Triangle Conjecture.

19. $m\widehat{KIM} = 2m\angle KEM = 140°$. Then $m\widehat{KI} = 140° - 70° = 70° = m\widehat{MI}$. Therefore $m\angle IKM = \frac{1}{2}m\widehat{MI} = \frac{1}{2}m\widehat{KI} = m\angle IMK$, so $\angle IKM \cong \angle IMK$. So $\triangle KIM$ is isosceles by the Converse of the Isosceles Triangle Conjecture.

LESSON 9.6

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EXERCISES

1. 456π cm^2. Look at the angles in quadrilateral $BODY$. By the Tangent Conjecture, $\angle OBY$ and $\angle ODY$ are right angles, so by the Quadrilateral Sum Conjecture, $90° + 105° + 90° + m\angle DOB = 360°$, and $m\angle DOB = 75°$. Because $\angle DOB$ is the central angle that intercepts $\widehat{BD}$ (the minor arc), $m\widehat{BD} = 75°$, and the measure of major arc BD is $360° - 75° = 285°$. Now find the area of the shaded region, which is a sector of the circle.

$$A = \left(\frac{a}{360°}\right)\pi r^2 = \left(\frac{285°}{360°}\right)\pi(24)^2$$
$$= \frac{19}{24}\pi(24)^2 = (19 \cdot 24)\pi = 456\pi \text{ cm}^2$$

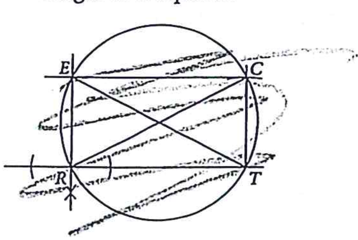
2. $(32\pi - 32\sqrt{3})$ cm^2. The area of the shaded region is the difference between the area of the semicircle and the area of the triangle. $\angle T$ is a right angle (Angles Inscribed in a Semicircle Conjecture), so $\triangle RTH$ is a 30°-60°-90° triangle in which $\overline{HT}$ is the longer leg (opposite the 60° angle), $\overline{RT}$ is the shorter leg (opposite the 30° angle), and $\overline{RH}$ is the hypotenuse. Because $HT = 8\sqrt{3}$, $RT = 8$, and $RH = 2(8) = 16$. $\overline{RH}$ is a diameter of the circle, so the radius is 8 cm.

Area of semicircle $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(8)^2 = \frac{1}{2}\pi 64 = 32\pi$ cm^2

Area of triangle $= \frac{1}{2}bh = \frac{1}{2}(8)(8\sqrt{3}) = 32\sqrt{3}$ cm^2

Area of shaded region $= (32\pi - 32\sqrt{3})$ cm^2

of the circumscribed circle. The circle's radius is the distance from the center to a vertex. It is not possible to construct the inscribed circle unless the rectangle is a square.



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12. C. The measure of $\angle ABP$ increases as it changes from an acute angle to a right angle to an obtuse angle. The other measures either increase and then decrease or do not change as P moves.

13. 38°. The conjecture discovered in Lesson 6.5, Exercise 16 and proved in Exercise 17 says that the measure of an angle formed by two intersecting chords is equal to one-half the sum of the intercepted arcs. Applying this conjecture, $a = \frac{1}{2}(32° + 44°) = \frac{1}{2}(76°) = 38°$.

14. 48°. By the Parallel Lines Intercepted Arcs Conjecture, the unmarked arc has measure b. Therefore $2b + 96° + 168° = 360°$, so $2b = 96°$, and $b = 48°$.

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13. 18°/sec. No, the angular velocity is the same at every point on the carousel.

$$\text{angular velocity} = \frac{360°}{20 \text{ sec}} = 18°/\text{sec}.$$

14. Outer horse $\approx$ 2.5 m/sec, inner horse $\approx$ 1.9 m/sec. One horse has traveled farther in the same amount of time (tangential velocity), but both horses have rotated the same number of times (angular velocity).

Calculation of tangential velocities:

Outer horse:

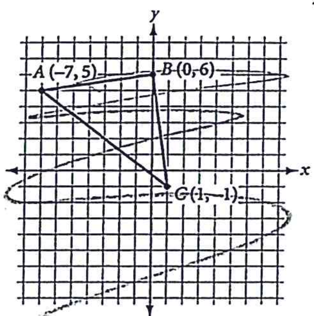
$C = 2\pi r = 2\pi(8) = 16\pi$ m

$$\text{tangential velocity} = \frac{\text{distance along circular path}}{\text{time}} = \frac{16\pi \text{ m}}{20 \text{ sec}} \approx 2.5 \text{ m/sec}$$

Inner horse:

$C = 2\pi r = 2\pi(6) = 12\pi$ m

$$\text{tangential velocity} = \frac{\text{distance along circular path}}{\text{time}} = \frac{12\pi \text{ m}}{20 \text{ sec}} \approx 1.9 \text{ m/sec}$$



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$\frac{5}{12}$ or 42%

It appears from the drawing that $\triangle ABC$ is a right triangle with $\angle B$ as the right angle. Verify this by finding slopes of the two sides of $\angle B$: Slope $\overline{AB} = \frac{6-5}{0-(-7)} = \frac{1}{7}$, and slope $\overline{BC} = \frac{-1-6}{1-0} = \frac{-7}{1} = -7$. Therefore $\overline{AB} \perp \overline{BC}$, so $\angle B$ is a right angle, and $\overline{AC}$ is the hypotenuse of the right triangle. Thus the