



Lesson 14.6

The Law of the Contrapositive



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"I do," Alice hastily replied: "at least—*at least* I mean what I
say—that's the same thing, you know."
"Not the same thing a bit," said the Hatter. "Why, you might
just as well say that 'I see what I eat' is the same thing as 'I eat
what I see!'"

—Alice's Adventures in Wonderland by Lewis Carroll

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Every conditional statement has three other conditionals associated with it. They are the converse, the inverse, and the contrapositive.

Statement: $P \rightarrow Q$
Converse: $Q \rightarrow P$
Inverse: $\sim P \rightarrow \sim Q$
Contrapositive: $\sim Q \rightarrow \sim P$

To create the converse of a conditional statement, the two parts of the statement are simply reversed. To create the inverse, the two parts are negated. To create the contrapositive, the two parts are reversed and negated.

In logic, conditional statements are either true or false. If a statement is true, is its converse necessarily true? Earlier in this text you worked with statements and converses, and you discovered that there is no true/false relationship that holds between a statement and its converse. How about its inverse? Contrapositive? Let's look at an example of a true conditional and its converse, its inverse, and its contrapositive.

Example

Statement: If two angles are vertical angles, then they are congruent.
($P \rightarrow Q$)
The statement is *true*.

Converse: If two angles are congruent, then they are vertical angles.
($Q \rightarrow P$)
The converse of the statement is *false*.
If two angles are not vertical angles, then they are not congruent.
($\sim P \rightarrow \sim Q$)
The inverse of the statement is *false*.

Contrapositive:

If two angles are not congruent, then they are not vertical angles.
($\sim Q \rightarrow \sim P$)

The contrapositive of the statement is *true*.

In this example, the contrapositive has the same truth value as the original conditional, and the converse and the inverse have the same truth value. Is this relationship always true? Let's investigate.

Investigation 14.6

Write the converse, the inverse, and the contrapositive for each of the four conditional statements below. Then identify each statement, converse, inverse, and contrapositive as true or false.

1. If it is a rose, then it is a flower.
2. If you're out of chocolate cake, you're out of dessert.
3. If the triangle is isosceles, then the triangle's base angles are congruent.
4. If $\triangle ABC$ is congruent to $\triangle DEF$, then $AB = DE$.

For each statement above, did the contrapositive have the same truth value as the original conditional? Did the inverse and the converse have the same truth value? This leads to our fourth form of logical reasoning, the law of the contrapositive.

The law of the contrapositive (LC) says that if a conditional statement is true, then its contrapositive is also true. Conversely, if the contrapositive is true, then the original conditional statement must also be true.

Law of the contrapositive

$$\begin{aligned} P &\rightarrow Q \\ \therefore \sim Q &\rightarrow \sim P \end{aligned}$$

The law of the contrapositive says that any conditional and its contrapositive are logically equivalent. You may replace one with the other.

From the investigation it should be clear that both a statement and its converse may or may not have the same truth value. However, a statement and its contrapositive always have the same truth value (LC). If one is true, then the other is true. If one is false, then the other is false. Look again at the inverse and the converse of one of the statements in the investigation. Notice that each is the contrapositive of the other. The inverse and the converse are logically equivalent. They are either both true or both false.

So far, you have learned four basic forms of valid reasoning.

Four forms of valid reasoning			
$P \rightarrow Q$	$P \rightarrow Q$	$P \rightarrow Q$	$P \rightarrow Q$
Converse:	$\sim Q \rightarrow \sim P$	$Q \rightarrow R$	$\therefore \sim Q \rightarrow \sim P$
Inverse:	$\therefore \sim Q$	$\therefore P \rightarrow R$	by LS
Contrapositive:	$\therefore \sim P$	by MP	by LC