Notes 6.4 - Vectors and Dot Products

HW#11a
p435: 2, 4, 13, 15, 16, 23

Def of dot product: If vector $u = \langle u_1, u_2 \rangle$ & $v = \langle v_1, v_2 \rangle$
then $u \cdot v = u_1 v_1 + u_2 v_2$

p429

(Think of this like basic multiplication of matrices $A_{1\times2} \cdot B_{2\times1} = AB_{1\times1}$

$$\begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = 3\cdot1 + 4\cdot5 = 23$$

In other words $u \cdot v$ is a number, a "scalar"

Ex) $u = \langle 3, -2 \rangle$ $v = \langle -1, 4 \rangle$

$u \cdot v = 3 \cdot -1 + -2 \cdot 4 = -11$

Properties: u, v & w are vectors in a plane or space; c is a scalar.

1) $u \cdot v = v \cdot u$ commutative
2) $0 \cdot v = 0$
3) $u \cdot (v + w) = u \cdot v + u \cdot w$
4) $v \cdot v = \|v\|^2$
5) $c(u \cdot v) = cu \cdot v$ OR $u \cdot cv$
(multiply only one of the vectors by the scalar)

Ex) $\langle 3,4 \rangle \cdot \langle 2,-3 \rangle$
$3\cdot2 + 4\cdot-3$
$6 - 12$
-6

$\langle -3,-5 \rangle \cdot \langle 1,-8 \rangle$
$-3\cdot1 + -5\cdot-8$
37

$\langle -6,5 \rangle \cdot \langle 5,6 \rangle$
$-6\cdot5 + 5\cdot6$
0
orthogonal vectors
(meet at a right $\angle$)

Using Properties (Ex 2 on p430)

$u = \langle 3,4 \rangle$ $v = \langle -2,6 \rangle$

$(u \cdot v)v$
$\langle 3,4 \rangle \cdot \langle -2,6 \rangle$
$-6 + 24$
18
$18 \cdot v = 18\langle -2,6 \rangle$
$\langle -36, 108 \rangle$
when you multiply by a scalar, you get a parallel vector
answer is a vector

$\begin{array}{r} 18 \\ \times 6 \\ \hline 108 \end{array}$

$u \cdot (u + v)$
$u + v = \langle 1, 10 \rangle$
$\langle 3,4 \rangle \cdot \langle 1,10 \rangle$
$3 + 40$
43
OR $u \cdot u + u \cdot v$
$3\cdot3 + 4\cdot4 + 3\cdot-2 + 4\cdot6$
$9 + 16 - 6 + 24$
43
answer is a scalar

$\|v\|$
$\sqrt{2^2 + 6^2}$
$\sqrt{40} = 2\sqrt{10}$