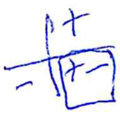
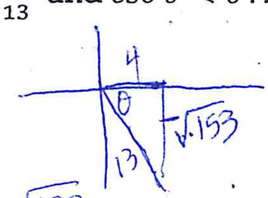
5.1 Notes

Triangle Problems: The Return

- You must know SOHCAHTOA
- You must know which side of the triangle will be negative based on which trig function is negative

Ex. Let $\cos\theta = \frac{4}{13}$ and $\csc\theta < 0$. Find the values of each of the following.





$4^2 + b^2 = 13^2$

$b^2 = 169 - 16$

$b^2 = 153$

$b = \sqrt{153}$

$\frac{opp}{hyp}$

1. $\sin\theta =$ $\frac{-\sqrt{153}}{13}$

2. $\sec\theta =$ $\frac{13}{4}$ $\frac{hyp}{adj}$

Notes about triangle problems:

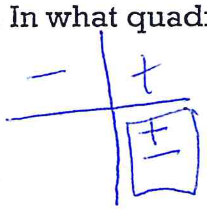
make sure you have selected correct quadrant based givens.
Sketch with $\triangle$.
Don't make mistakes with PYTHAGOREAN THEOREM.

How to find the quadrant a certain trig function is in:

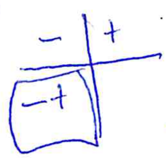
1. Get the trig function in terms of $\sin\theta$ or $\cos\theta$
2. Find out where each trig function is positive or negative
3. Find where the two overlap in a quadrant

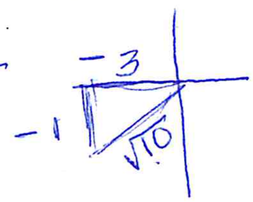
III $\tan x = \frac{1}{3}$ $\cos x < 0$

Ex. In what quadrant is $\tan\theta < 0$ and $\cos\theta > 0$



Quad IV $(+, -)$
adj, opp





$\tan = \frac{opp}{adj}$

$hyp = \sqrt{1^2 + 3^2}$

$\sqrt{10}$

$\cot x = 3$

$\cos x = \frac{adj}{hyp} = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

$\sin x = \frac{opp}{hyp} = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10}$

Simplifying using factoring

- Be on the lookout for difference of squares, GCF, and AC methods factoring

Let $\sin\theta = x$

1. $16\csc^2\theta - 49 =$

$(4\csc\theta + 7)(4\csc\theta - 7)$

2. $15 + 8\sin\theta + \sin^2\theta =$

$15 + 8x + x^2$

$(x+5)(x+3) =$

$(\sin\theta + 5)(\sin\theta + 3)$

5.1 Notes

Simplifying using factoring AND fundamental identities

- Know your identities!
  - Look to get things in terms of $\sin\theta$ or $\cos\theta$, or of the other side of the equation
  - Use Pythagorean identities to help substitute for expressions
- **IMPORTANT**: You cannot perform equivalent operations to both sides of a trig identity like you could in solving an equation

even $=\cos(\theta)$

1. $\tan^2\theta\cos\theta + \cos\theta$

$\frac{\sin^2\theta\cos\theta}{\cos^2\theta} + \cos\theta$

$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \boxed{\sec\theta}$

2. $\boxed{\sec\theta}\cos(-\theta) - \sin^2\theta$

$\frac{\cos(-\theta)}{\cos\theta} - \sin^2\theta$

$1 - \sin^2\theta$

$\boxed{\cos^2\theta}$

Verifying algebraically

- Use the same methods as you would with simplification
- **IMPORTANT**: You **cannot** perform equivalent operations to both sides of a trig identity like you could in solving an equation (i.e. you can't cross the = sign)

Ex. Verify algebraically in the vertical format

1. $\frac{\tan\theta + \tan\left(\frac{\pi}{2}-\theta\right)}{\sec(-\theta)} = \csc\theta$

$\frac{\tan\theta + \cot\theta}{\sec\theta} = \csc\theta$

2. $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2\tan\theta$

Factor

$4\tan^2\theta + \tan\theta - 3$

$(4\tan\theta - 3)(\tan\theta + 1)$

$\frac{4}{4}$ -3

Ex) Adding Trig Expressions

$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$(1+\sin\theta)(1-\sin\theta)$

$1-\sin^2\theta$

$\frac{1-\sin\theta}{1-\sin^2\theta} + \frac{1+\sin\theta}{1-\sin^2\theta}$

$\frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} =$

$\boxed{2\sec^2\theta}$

$\sec^2x + 3\tan x + 1$

$1 + \tan^2x + 3\tan x + 1$

$\tan^2x + 3\tan x + 2$

$(\tan x + 2)(\tan x + 1)$

Trigonometric Substitution

1. Plug the value in for x
2. Simplify using trigonometric identities

Ex. Use trigonometric substitution to write the algebraic expression as a trig function of θ, where $0 < \theta < \frac{\pi}{2}$. Use $x = 3\sin\theta$

$\sin^2\theta = (\sin\theta)^2$

$\sqrt{9-x^2} =$

$\sqrt{9-(3\sin\theta)^2}$

$\sqrt{9-9\sin^2\theta} = \sqrt{9\underbrace{(1-\sin^2\theta)}_{\text{use Pyth. Id}}} = \sqrt{9\cdot\cos^2\theta} = 3\cos\theta$