

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp (\tan u)(\tan v)}$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

Double-Angle Formulas

$$\sin 2x = \frac{2 \sin x \cos x}{1}$$

Use  $\sin(u+v) = \sin u \cos v + \sin v \cos u$   
 $\sin(x+x) = 2 \sin x \cos x$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\tan(x+x) = \frac{\tan x + \tan x}{1 - \tan^2 x}$$

$$\cos 2x = \frac{\cos^2 x - \sin^2 x}{1}$$

$\cos(x+x) = \cos x \cos x - \sin x \sin x =$

$$\cos 2x = \frac{2 \cos^2 x - 1}{1}$$

$$\cos 2x = \frac{1 - 2 \sin^2 x}{1}$$

Use  $\cos^2 x + \sin^2 x = 1$   
to manipulate

Power-Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

rearrange last formula for  $\cos 2x$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

same as  $\sin^2 x$   
only opposite  
 ~~$\cos^2 x = \frac{1 - \cos 2x}{2}$~~   
 ~~$\cos 2x = 1 - 2 \cos^2 x$~~

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x} \quad \frac{\sin^2 x}{\cos^2 x}$$

Half-Angle Formulas

$$\sin \frac{x}{2} = \frac{\pm \sqrt{1 - \cos x}}{2}$$

$$\cos \frac{x}{2} = \frac{\pm \sqrt{1 + \cos x}}{2}$$

$\sqrt{\sin^2 \left(\frac{x}{2}\right) = \frac{1 - \cos 2\left(\frac{x}{2}\right)}{2} = \frac{1 - \cos x}{2}}$

$$\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} \quad \text{or} \quad \frac{\sin x}{1 + \cos x}$$

The signs of  $\sin \frac{x}{2}$  and  $\cos \frac{x}{2}$  depend on the quadrant in which  $\frac{x}{2}$  lies.