$BG = 27$ ft

$BH = 24$ ft

$CF = y$

$DE = x$

To solve for y, use $\triangle CBG$.

$\tan 84° = \dfrac{27}{\frac{y}{2}}$

$\frac{y}{2}\tan 84° = 27$

$y = \frac{54}{\tan 84°} \approx 5.7$ ft

To solve for x, use $\triangle EBH$. Since $\overleftrightarrow{CF}$ and $\overleftrightarrow{DE}$ contain the bases of a trapezoid, they are parallel. Therefore $\angle HEB \cong \angle GFB$ by Corresponding Angles Conjecture.

$\tan 84° = \dfrac{24}{\frac{x}{2}}$

$\frac{x}{2}\tan 84° = 24$

$x = \frac{48}{\tan 84°} \approx 5.1$ ft

For z:

$CD = BC - BD$

$\sin 84° = \frac{27}{BC}$, so $BC = \frac{27}{\sin 84°}$, or approximately 27.17 ft.

$\sin 84° = \frac{24}{BD}$, so $BD = \frac{24}{\sin 84°}$, or approximately 24.13 ft.

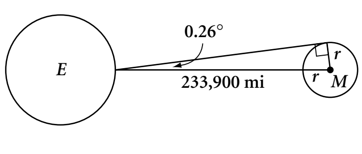
$CD = 27.15 - 24.13 \approx 3.02$ ft

Developing Mathematical Reasoning

1	6	2	4	0	3	2	0
1	5	3	1	5	1	0	4
6	0	3	4	3	6	3	5
2	4	2	2	0	4	5	2
1	0	5	0	0	6	5	3
4	5	3	6	2	3	4	6
6	1	2	1	5	1	4	6

Extensions

Make a sketch.



Use the tangent ratio to find the radius.

$$\tan 0.26° = \frac{r}{r + 233{,}900}$$

$$(r + 233{,}900)\tan 0.26° = r$$

$$r\tan 0.26° + 233{,}900\tan 0.26° = r$$

$$r\tan 0.26° + 233{,}900\tan 0.26° - r = 0$$

$$r(\tan 0.26° - 1) = -233{,}900\tan 0.26°$$

$$r = \frac{-233{,}900\tan 0.26°}{\tan 0.26° - 1} \approx 1{,}070 \text{ m}$$

Exploration: Three Types of Proof

Developing Mathematical Reasoning

Students might make a table or see recursively how many pieces are added by each cut (and what the number is after the first cut).

1. 151 **2.** 100 **3.** 200

Chapter 12 Review

Exercises

1. 0.8387

2. 0.9877

3. 28.6363

4. $\sin A = \frac{a}{b}$; $\cos A = \frac{c}{b}$; $\tan A = \frac{a}{c}$.

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}} = \frac{a}{b}$$

$$\cos A = \frac{\text{length of side adjacent of } \angle A}{\text{length of hypotenuse}} = \frac{c}{b}$$

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A} = \frac{a}{c}$$

5. $\sin B = \frac{8}{17}$; $\cos B = \frac{15}{17}$; $\tan B = \frac{8}{15}$. The side lengths in this triangle are a multiple of the 8-15-17 Pythagorean triple: $16 = 2(8)$, $30 = 2(15)$, so $OB = 2(17) = 34$. The trigonometric ratios are the same in this triangle as in an 8-15-17 triangle.

6. $\sin\theta = s$; $\cos\theta = t$; $\tan\theta = \frac{s}{t}$. Draw a perpendicular segment from the point (t, s) to the x-axis to form a right triangle. In this triangle, the length of the horizontal leg will be t, the length of the vertical length will be s, and the length of the hypotenuse will be 1, the radius of the circle. Then $\sin\theta = \frac{s}{1} = s$, $\cos\theta = \frac{t}{1} = t$, and $\tan\theta = \frac{s}{t}$.

7. Several approaches are possible. Here is one of the simplest.

$$\frac{a}{\sin A} = \frac{1}{\frac{\sin A}{a}} = \frac{1}{\frac{\sin B}{b}} = \frac{b}{\sin B}$$

A similar argument can be used for $\frac{c}{\sin C}$.

8. $a^2 = b^2 + c^2 - 2bc\cos A$; $b^2 = a^2 + c^2 - 2ac\cos B$

9. 33°. $A = \sin^{-1}(0.5447) \approx 33°$.

10. 86°. $B = \cos^{-1}(0.0696) \approx 86°$.

11. 71°. $C = \tan^{-1}(2.9043) \approx 71°$.

12. 1823 cm². The shaded region is a semicircle; you need to know the radius of the circle to find its area. By the Angles Inscribed in a Semicircle Conjecture, the triangle in the figure is a right triangle whose hypotenuse is the diameter of the circle that is drawn in the figure. Let c equal the length of the hypotenuse.

$$\sin 37° = \frac{41}{c}$$
$$c(\sin 37°) = 41$$
$$c = \frac{41}{\sin 37°}$$

(Store this value in your calculator rather than rounding this intermediate result.)

Let r represent the radius of the circle. Then $r = \frac{1}{2}c$, and the area of the semicircle is given by $A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{1}{2} \cdot \frac{41}{\sin 37°}\right)^2 \approx 1823 \text{ cm}^2$.

13. 15,116 cm³. Look at the right triangle inside the cone. The shorter leg of this triangle is a radius of the circle, and the longer leg is the height of the cone. The angle with measure 112° is an exterior angle of the triangle. Its supplement, which is the larger acute angle in the triangle, measures $180° - 112° = 68°$. Use the triangle to find h.

$$\tan 68° = \frac{h}{18}$$
$$h = 18(\tan 68°)$$

(Store this value in your calculator rather than rounding this intermediate result.)

Now find the volume of the cone.

$$V = \frac{1}{3}BH = \frac{1}{3}\pi r^2 H = \frac{1}{3}\pi(18)^2(18 \tan 68°)$$
$$\approx 15{,}116 \text{ cm}^3$$

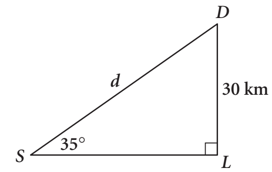
14. Yes, the plan meets the act's requirements. The angle of ascent is approximately 4.3°. Use the side view to find the slope of the ramp.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1.5}{20} = 0.075$$

The slope must be less than $\frac{1}{12} = 0.08\overline{3}$. Because $0.075 < 0.08\overline{3}$, the ramp is not too steep. By looking at the top and front views, you can see that there is a 5-by-5 ft landing for every 1.5 ft of rise, so the landing requirement is exceeded. Thus, all requirements of the Americans with Disabilities Act are met.

The angle of ascent is $\tan^{-1}\left(\frac{1.5}{20}\right) \approx 4.3°$.

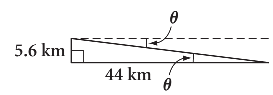
15. Approximately 52 km. Let d represent the distance between the sailboat and the dock. In the figure below, S represents the position of the sailboat, L represents the position of the lighthouse, and D represents the position of the dock.



$$\sin 35° = \frac{30}{d}$$
$$d(\sin 35°) = 30$$
$$d = \frac{30}{\sin 35°} \approx 52$$

The distance between the lighthouse and the dock is approximately 52 km.

16. Approximately 7.3°. Let θ be the angle of descent. Recall that the angle of descent (or angle of depression) is measured with respect to the horizontal. Make a sketch.



By the AIA Conjecture, the smaller acute angle in the right triangle is congruent to the angle of descent. Use this triangle to find θ.

$$\tan \theta = \frac{5.6}{44}$$
$$\theta = \tan^{-1}\left(\frac{5.6}{44}\right) \approx 7.3°$$

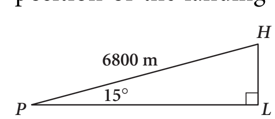
The angle of descent is approximately 7.3°.

17. Approximately 22 ft. Let l represent the length of one rafter, not including the overhang. The center line of the house is the altitude from the vertex angle of the isosceles triangle, so it is also a median. Look at the right triangle on the left, which has longer leg of length $\frac{1}{2}(32) = 16$ ft and hypotenuse of length l.

$$\cos 36° = \frac{16}{l}$$
$$l(\cos 36°) = 16$$
$$l = \frac{16}{\cos 36°} \approx 20 \text{ ft}$$

Add 2 ft for the overhang. The carpenter should make each rafter approximately 22 ft long.

18. Approximately 6568 m. In the figure below, P represents the position of the patrol boat, H represents the position of the helicopter, and L represents the position of the landing spot.



$$\cos 15° = \frac{PL}{6800}$$
$$PL = 6800(\cos 15°) \approx 6568$$

The patrol boat is approximately 6568 m from the point where the package will land.