p490-492 p485:15: π, 2π

With the right-angled corner of a carpenter's square: Place the corner in the circle so that it is an inscribed right angle. Trace the sides of the corner. Use the square to construct the hypotenuse of the right triangle (which is the diameter of the circle). Repeat. The center is the intersection of the two diameters.

3. The velocity vector is always perpendicular to the radius at the point of tangency to the object's circular path.

4. Sample answer: An arc measure is between 0° and 360°. An arc length is proportional to arc measure and depends on the radius of the circle.

5. 55°. Draw the radius to the point of tangency. By the Tangent Conjecture, the radius is perpendicular to the tangent, so a right triangle has been formed. Therefore the measure of the central angle that intercepts the arc of measure b is $90° - 35° = 55°$, so $b = 55°$ by the definition of the measure of an arc.

6. 65°. The 110° angle is an inscribed angle that intercepts an arc of measure $a + 155°$, so by the Inscribed Angle Conjecture, $110° = \frac{1}{2}(a + 155°)$; $220° = a + 155°$, and $a = 65°$.

7. 128°. Congruent chords intercept congruent arcs (Chord Arcs Conjecture), so the unmarked arc has measure c. Then $2c + 104° = 360°$, so $2c = 256°$, and $c = 128°$.

8. 118°. First find the measure of either of the two vertical angles that form a linear pair with the angle of measure e. The measure of an angle formed by two intersecting chords is half the sum of the measures of the intercepted arcs (Intersecting Chords Conjecture), so the measure of either one of these angles is $\frac{1}{2}(60° + 64°) = 62°$. Then $e = 180° - 62°$ (Linear Pair Conjecture), so $e = 118°$.

9. 91°. The angle marked as a right angle intercepts a semicircle and is therefore inscribed in the opposite semicircle, so $d + 89° = 180°$, and $d = 91°$. (You could also use the Cyclic Quadrilateral Conjecture to see that the angle opposite the marked right angle is the supplement of a right angle, and therefore, it is also a right angle. Then the intercepted arc of this right angle must measure 180°, so $d + 89° = 180°$.)

10. 66°. Look at either of the angles that form a linear pair with the 88° angle. The supplement of an 88° angle is a 92° angle. Because the measure of an angle formed by two intersecting chords is one-half the sum of the measures of their intercepted arcs (Intersecting Chords Conjecture), $92° = \frac{1}{2}(f + 118°)$, so $184° = f + 118°$, and $f = 66°$.

11. 125.7 cm. $C = 2\pi r = 2\pi(20) \approx 125.7$ cm.

12. 42.0 cm. $C = \pi d$, so $132 = \pi d$, and $d = \frac{132}{\pi} \approx$ 42.0 cm.

13. 15π cm. $m\widehat{AB} = 100°$ (Chord Arcs Conjecture), so by the Arc Length Conjecture, the length of $\widehat{AB}$ is $\frac{100°}{360°}C = \frac{5}{18}(2\pi \cdot 27) = 15\pi$ cm.

14. 14π ft. By the Intersecting Secants Conjecture, $50° = \frac{1}{2}(m\widehat{DL} - 60°)$, so $100° = m\widehat{DL} - 60°$, and $m\widehat{DL} = 160°$. By the Chord Arcs Conjecture, $m\widehat{CD} = m\widehat{OL}$, so $2m\widehat{CD} + 160° + 60° = 360°$. Then $2m\widehat{CD} = 140°$, and $m\widehat{CD} = 70°$. Now apply the Arc Length Conjecture. The length of $\widehat{CD}$ is $\frac{70°}{360°}C = \frac{7}{36}(2\pi \cdot 36 \text{ ft}) = 14\pi$ ft.

15. Look at the inscribed angles with measures 57° and 35°. By the Inscribed Angle Conjecture, the sum of the measures of their intercepted arcs is $2(57°) + 2(35°) = 184°$. However, the sum of these two arcs is a semicircle, and the measure of a semicircle is 180°, so this is impossible.

Another way to look at this is to add the angle measures in the triangle. The third angle of the triangle must be a right angle because it is inscribed in a semicircle (Angles Inscribed in a Semicircle Conjecture). Therefore, the sum of the three angles of the triangle would be $57° + 35° + 90° = 182°$, which is impossible by the Triangle Sum Conjecture.

16. By the Parallel Lines Intercepted Arcs Conjecture, the unmarked arc measures 56°. Then the sum of the arcs would be $84° + 56° + 56° + 158° = 354°$, but this is impossible because the sum of the measures of the arcs of a circle must be 360°.

17. $m\angle EKL = \frac{1}{2}m\widehat{EL} = \frac{1}{2}(180° - 108°) = 36° = m\angle KLY$. Therefore, $\overline{KE} \parallel \overline{YL}$ by the Converse of the Parallel Lines Conjecture.

18. $m\widehat{JI} = 360° - 56° - 152° = 152° = m\widehat{MI}$. Therefore, $m\angle J = m\angle M$ (Inscribed Angles Intercepting Arcs Conjecture), and $\triangle JIM$ is isosceles by the Converse of the Isosceles Triangle Conjecture.

19. $m\widehat{KIM} = 2m\angle KEM = 140°$. Then $m\widehat{KI} = 140° - 70° = 70° = m\widehat{MI}$. Therefore, $m\angle IKM = \frac{1}{2}m\widehat{MI} = \frac{1}{2}m\widehat{KI} = m\angle IMK$, so $\angle IKM \cong \angle IMK$. So, $\triangle KIM$ is isosceles by the Converse of the Isosceles Triangle Conjecture.

20. Ertha can trace the incomplete circle on paper. She can lay the corner of the pad on the circle to trace an inscribed right angle. Then Ertha should mark the endpoints of the intercepted arc and use the pad to construct the hypotenuse of the right triangle, which is the diameter of the circle.

25 a) $\frac{\pi}{3}$ b) π c) $\frac{3\pi}{2}$ d) $\frac{7\pi}{4}$

28. Melanie: 151 m/min or 9 km/h; Melody: 94 m/min or 6 km/h. For Melanie, the radius is 8 m, so the circumference (or length of one revolution) is 16π m, and for Melody, the radius is 5 m, so the circumference is 10π m.

The merry-go-round makes 30 revolutions in 10 minutes, or 3 revolutions/minute. Find the average speeds for Melanie and Melody in kilometers per hour.

Melanie:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{3C}{1\text{ min}}$$
$$= \frac{3(16\pi\text{ m})}{1\text{ min}} \approx 151\text{ m/min}$$
$$= \frac{3(16\pi\text{ m})}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ h}} \cdot \frac{0.001\text{ km}}{1\text{ m}}$$
$$\approx 9\text{ km/h}$$

LESSON 10.4 p 522

EXERCISES

1. $18\pi\text{ cm}^2 \approx 56.5\text{ cm}^2$. First find the length of the shorter leg of the right triangle. Because $16 = 4 \cdot 4$ and $20 = 5 \cdot 4$, the side lengths are a multiple of the 3-4-5 Pythagorean triple, and the length of the shorter leg is $3 \cdot 4 = 12$ cm. The shorter leg of the right triangle is also a diameter of the shaded semicircle, so the radius is $\frac{1}{2}(12) = 6$ cm. Therefore, the area of the shaded region is $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(6)^2 = \frac{1}{2}\pi(36) = 18\pi\text{ cm}^2 \approx 56.5\text{ cm}^2$.

4. $120\pi\text{ cm}^2 \approx 377.0\text{ cm}^2$. $\overrightarrow{TA}$ is tangent to the circle N at A, so $\overrightarrow{TA} \perp \overline{NA}$ by the Tangent Conjecture. Then $\angle TAN$ is a right angle and $\triangle TAN$ is a 30°-60°-90° triangle. The length of the longer leg, $\overline{TA}$, is $12\sqrt{3}$ cm, so the length of the shorter leg, $\overline{NA}$, is 12 cm. Because $\overline{NA}$ is also a radius of the circle, the area of the circle is $144\pi\text{ cm}^2$. The area of the shaded region is $\frac{360-60}{360}$, or $\frac{5}{6}$, of the area of the circle. Therefore, the shaded area is $\frac{5}{6}(144\pi)$, or $120\pi\text{ cm}^2 \approx 377.0\text{ cm}^2$.

5. $(32\pi - 32\sqrt{3})\text{ cm}^2 \approx 45.1\text{ cm}^2$. The area of the shaded region is the difference between the area of the semicircle and the area of the triangle. $\angle T$ is a right angle (Angles Inscribed in a Semicircle Conjecture), so $\triangle RTH$ is a 30°-60°-90° triangle in which $\overline{HT}$ is the longer leg (opposite the 60° angle), $\overline{RT}$ is the shorter leg (opposite the 30° angle), and $\overline{RH}$ is the hypotenuse. Because $HT = 8\sqrt{3}$, $RT = 8$, and $RH = 2(8) = 16$ cm. $\overline{RH}$ is a diameter of the circle, so the radius is 8 cm.

Area of semicircle $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(8)^2 = \frac{1}{2}\pi 64 = 32\pi\text{ cm}^2$

Area of triangle $= \frac{1}{2}bh = \frac{1}{2}(8)(8\sqrt{3}) = 32\sqrt{3}\text{ cm}^2$

Area of shaded region $= (32\pi - 32\sqrt{3})\text{ cm}^2 \approx 45.1\text{ cm}^2$