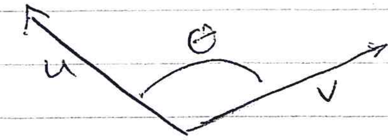
More notes 6.4 p 430

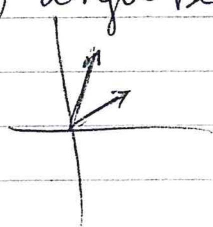


$0 < \theta < 180$ $0 < \theta < \pi$

angle between 2 nonzero vectors

$\cos\theta = \dfrac{u \cdot v}{\|u\| \, \|v\|}$ $\dfrac{\text{dot product}}{\text{product of magnitudes}} = \cos\theta$

Ex) angle between $u\langle 2,1\rangle$ $v\langle 1,3\rangle$



$2 \cdot 1 + 1 \cdot 3 = 5$

$\sqrt{2^2+1^2} \cdot \sqrt{1^2+3^2} = \sqrt{50}$

$\cos^{-1}\left(\dfrac{5}{\sqrt{50}}\right) = 45^\circ$

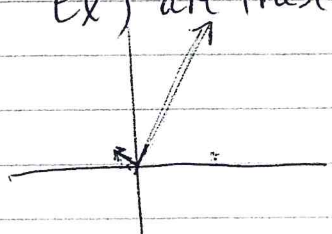
Orthogonal vectors are $\perp$; their dot product $= 0$

$u \cdot v = 0$

$\cos\theta = 0$ $\theta = \frac{\pi}{2}$ or 90°

a zero vector is orthogonal to all vectors because $0 \cdot v = 0$

Ex) are these orthogonal? $u = \langle 6, 10\rangle$ $v = \langle -\frac{1}{3}, \frac{1}{5}\rangle$



yes

$6 \cdot -\frac{1}{3} + 10 \cdot \frac{1}{5}$

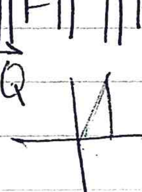
$-2 + 2 = 0$

Work = magnitude of force · distance

if angle if not "collinear" with motion

$w = \cos\theta \, \|F\| \, \|\overrightarrow{PQ}\|$

$= F \cdot \overrightarrow{PQ}$



$\cos 60^\circ \cdot 50 \cdot 12$

$\frac{1}{2} \cdot 50 \cdot 12$

300 ft lbs

Ex) $\cos 30^\circ \cdot 35 \cdot 40$

1212.436 ft lb

HW#11a p435: 2, 4, 13, 15, 16, 23

2) $u_1 v_1 + u_2 v_2$

4) orthogonal ($\perp$)

13) $u \cdot u = \langle 3,3 \rangle \cdot \langle 3,3 \rangle$

scalar $3 \cdot 3 + 3 \cdot 3 = 18$

15) $(u \cdot v)v$

$\langle 3,3 \rangle \cdot \langle -4,2 \rangle$ vector

$3 \cdot -4 + 3 \cdot 2 = -6\langle -4,2 \rangle = \underline{\langle 24,-12 \rangle}$

16) $(u \cdot 2v)w$

$2v = \langle -8,4 \rangle$

$(\langle 3,3 \rangle \cdot \langle -8,4 \rangle)w$

$3 \cdot -8 + 3 \cdot 4 = -12\langle 3,-1 \rangle$

vector $\langle -36, 12 \rangle$

23) $\|u\|^2 = u \cdot u$ so $\|u\| = \sqrt{u \cdot u}$

$\sqrt{8^2+15^2}$

$\boxed{17}$

or $\sqrt{\langle -8,15 \rangle \cdot \langle -8,15 \rangle}$

$\sqrt{64+225}$

$\sqrt{289} = 17$

11b

31) $\cos\theta = \dfrac{u \cdot v}{\|u\|\|v\|} =$

$u = \langle 3,4 \rangle$ $v = \langle 0,-2 \rangle$

$\dfrac{-8}{5 \cdot 2} = \dfrac{-8}{10} = \dfrac{-4}{5}$

$\cos\theta = \frac{-4}{5}$ in radians

≈ 2.498

47) $\cancel{\cos\theta}$

$\cos\frac{2\pi}{3} = \dfrac{u \cdot v}{4 \cdot 10}$ $-\frac{1}{2}, \frac{\sqrt{3}}{2}$

$40 \cdot \cos\frac{2\pi}{3}$

-20 $-\frac{1}{2} \cdot 40 = -20$

53) ~~⟨2,2⟩~~

$\langle 2,-2 \rangle \cdot \langle -1,-1 \rangle$

$-2+2=0$

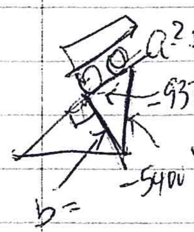
orthogonal

55) $\langle \frac{3}{4}, \frac{-1}{4} \rangle \cdot \langle 5,6 \rangle$

$\frac{15}{4} + \frac{-6}{4} = \frac{9}{4} \neq 0$

74) Force $\langle 0,-5400 \rangle \cdot \langle \cos 10, \sin 10 \rangle$

$-5400 \cdot \sin 10 \approx 937.7$ lb



$a^2+b^2=c^2$

$(-5400)^2 = (-937.7)^2 + b^2$

$\sqrt{5400^2 - 937.7^2} = b^2$

≈ 5318 lbs

77) $\cos 30° \cdot 45 \cdot 20$

$\frac{\sqrt{3}}{2} \cdot 900$

$450\sqrt{3} \approx 779.4$ ft lbs

81) $\cos 20° \cdot 25 \cdot 50$

$1250 \cos 20°$

≈ 1174.62 ft-lbs