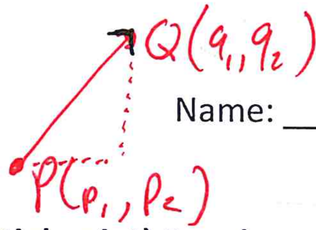
Precal – Section 6.3 Notes Vectors Name: ____________________

Completed Notes at: http://bit.ly/2E2XpPw



Each vector has a starting point (called the initial point) P and an ending point (called the terminal point) Q. P gives values (p1, p2) and Q gives values (q1, q2). Writing a vector in component form means you are breaking it down to its horizontal component (how far left or right) and vertical component (how much up or down), and can be found with this formula: $\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle$. The magnitude ($\|v\|$) of a vector is the same as asking the length of the vector, and can be found by the formula: $\|v\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$ but we usually use Pythagorean Theorem.

distance formula

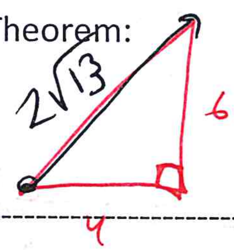
1. Sketch the vector with an initial point at (-1,-1) and a terminal point at (3, 5):

2. Find the component form of the vector by using the formula above, and comparing it to just counting the distance traveled <how far left or right, how far up or down>:

$\langle 3 - -1, 5 - -1 \rangle$

$\langle 4, 6 \rangle$

3. Find the magnitude of the vector using the formula above, and sketching in a right triangle to use Pythagorean Theorem:



$C = \sqrt{16 + 36}$

$C = \sqrt{52}$

$C = 2\sqrt{13}$

52 → 2, 26 → 2, 13

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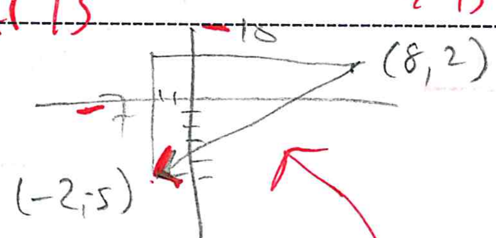
4. Sketch the vector with an initial point at (8,2) and a terminal point at (-2, -5):



5. Find the component form of the vector by using the formula above, and comparing it to just counting the distance traveled <left or right, up or down>: $\langle -10, -7 \rangle$

6. Find the magnitude of the vector using the formula above, and sketching in a right triangle to use Pythagorean Theorem:

$\sqrt{10^2 + 7^2}$

$\boxed{\sqrt{149}}$

$2\langle -2, 5\rangle$

You can also perform basic arithmetic operations with vectors:

If $v = \langle -2, 5\rangle$ and $w = \langle 3, 4\rangle$ then,

7. $2v =$ $\langle -4, 10\rangle$

8. $w - v =$ $\langle 5, -1\rangle$

If $v = \langle 1, -4\rangle$ and $w = \langle -3, 8\rangle$ then,

$\langle -6, 16\rangle + \langle 1, -4\rangle$

9. $\frac{1}{2}w =$ $\langle -3/2, 4\rangle$

10. $2w + v =$ $\langle -5, 12\rangle$

11. If the magnitude is 1 we call it a unit vector.

Often times we need to change a vector into a unit vector (but stay in the same direction) and this can be achieved by simply dividing the vector by its magnitude.

12. $< -3, 4 >$ What is the magnitude of this vector? 5

13. How can we get this to be a unit vector? $\div 5$

14. Given the vector $< -3, 4 >$, find another vector that is in the same direction but only has a length of 1 (a magnitude of 1): $\frac{\langle -3, 4\rangle}{5} \rightarrow \langle \frac{-3}{5}, \frac{4}{5}\rangle$

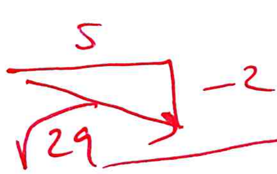
Unit vectors are a useful manipulation tool. We can multiply a desired magnitude by a unit vector in order to keep the original direction but with the wanted length:

15. Given the vector $< -3, 4 >$, find another vector that is in the same direction but only has a length of 7: $7\langle \frac{-3}{5}, \frac{4}{5}\rangle \rightarrow \boxed{\langle \frac{-21}{5}, \frac{28}{5}\rangle}$

16. Find another vector that is in the same direction of $< 5, -2 >$, but only has a length of 12: ______

Unit vector: $\div \|v\|$

$\div \sqrt{29}$

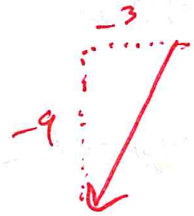


$\langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}}\rangle$

$\boxed{\langle \frac{60\sqrt{29}}{29}, \frac{-24\sqrt{29}}{29}\rangle}$ $12 \cdot \langle \frac{5\sqrt{29}}{29}, \frac{-2\sqrt{29}}{29}\rangle$ (1)

17. Find another vector that is in the same direction of $< -3, -9 >$,

but only has a length of 5: ________



$\|v\| = \sqrt{81+9}$

$= \sqrt{90} = 3\sqrt{10}$

$\left\langle \frac{-5\sqrt{10}}{10}, \frac{-15\sqrt{10}}{10} \right\rangle$

$\boxed{\left\langle -\frac{\sqrt{10}}{2}, \frac{-3\sqrt{10}}{2} \right\rangle}$

Unit Vector: $\left\langle \frac{-3}{3\sqrt{10}}, \frac{-9}{3\sqrt{10}} \right\rangle$

$\left\langle \frac{-\sqrt{10}}{10}, \frac{-3\sqrt{10}}{10} \right\rangle$

~~18. Find another vector that is in the same direction of $< 8, 6 >$,~~

~~but only has a length of 2: ________~~

~~**ONE TOO MANY PRACTICE PROBLEMS**~~

Vectors can be written with the arrows: < , > or with the letters i and j.

19. The vector $< -5, 3 >$ can also be written like: $-5i + 3j$

20. The vector $< 3, -4 >$ can also be written like: $3i - 4j$

21. The vector 5i + 4j can also be written like: $\langle 5, 4 \rangle$

22. The vector i – 3j can also be written like: $\langle 1, -3 \rangle$

If you prefer one form over the other, just switch it from the beginning of the problem.

An alternative to Component Form is Trigonometric Form.

23. Benefits of Component Form:
- Know how far left or right.
- Know how far up or down.

$v = < -5, 9 >$:

24. *Horizontal Displacement*: -5 down 5

25. *Vertical Displacement*: 9 up 9

$v = \langle 8, -2 \rangle$:

26. *Horizontal Displacement:* Right 8 27. *Vertical Displacement:* down 2

28. Benefits of Trigonometric Form:

- Quickly find the magnitude
- Directional Angle

$v = 5(\cos 46°i + \sin 46°j)$:

29. $||v||$ = 5 30. Directional Angle: 46°

$v = 8(\cos 135° i + \sin 135° j)$:

31. $||v||$ = 8 32. Directional Angle: 135°

Since Component Form and Trigonometric Form both have some benefits to each one, we need to be able to convert between each of them:

33. Find the component form of **v** given a magnitude of 5 and the angle it makes with the positive x-axis of 30°:

Because we are given the directional angle and the magnitude, we should start with trigonometric form: $5(\cos 30° i + \sin 30° j) \rightarrow 5 \text{cis } 30°$

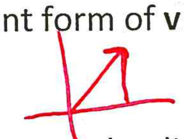
Distribute the magnitude, and then evaluate the trig functions using the unit circle (or your calculator, if it isn't an angle from the unit circle):

$\langle 5\cos 30°, 5\sin 30° \rangle$

(Optional) Change from the i and j format to the arrow < , > format.

$\langle \frac{5\sqrt{3}}{2}, 5/2 \rangle$

33. Find the component form of **v** given a magnitude of 8 and the angle it makes with the positive x-axis of 45°:



Because we are given the directional angle and the magnitude, we should start with trigonometric form:

$8 \text{cis } 45°$

$8(\cos 45° i + \sin 45° j)$

Distribute the magnitude, and then evaluate the trig functions using the unit circle (or your calculator, if it isn't an angle from the unit circle):

$8\cos 45° i + 8 \sin 45° j$

$4\sqrt{2} i + 4\sqrt{2} j$

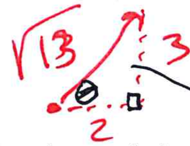
(Optional) Change from the i and j format to the arrow < , > format.

$\langle 4\sqrt{2},\ 4\sqrt{2} \rangle$

33. Find the trigonometric form of **v** given the component form of $v = \langle 2, 3 \rangle$:

Because we are given the component form we can easily start with a picture:

$\sqrt{4+4}$



$\tan^{-1}\left(\frac{3}{2}\right) = 56.310^\circ$

Trigonometric Form needs the magnitude and the directional angle so we need to calculate each of those:

$\sqrt{13}$

$\sqrt{13}\left(\cos 56.310^\circ i + \sin 56.310^\circ j\right)$

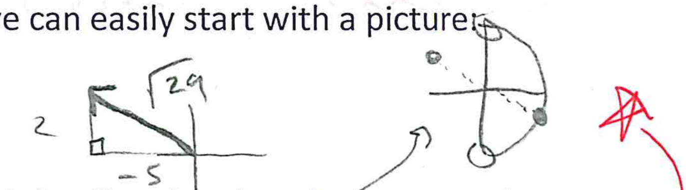
$\sqrt{13}\ \text{cis}\ 51.310^\circ$

Plug the numbers into their corresponding places in trigonometric form. Use the i and j format or the < , > format (whichever you prefer).

$\langle \sqrt{13}\cos 51.310^\circ,\ \sqrt{13}\sin 56.310^\circ \rangle$

33. Find the trigonometric form of **v** given the component form of $v = \langle -5,\ 2 \rangle$:

Because we are given the component form we can easily start with a picture:



Trigonometric Form needs the magnitude and the directional angle so we need to calculate each of those:

$\sqrt{4+25}$

$\sqrt{29}$

$\theta = \tan^{-1}\left(\frac{2}{-5}\right) = -21.801^\circ$

$+180^\circ$

158.199°

$\sqrt{29}\ \text{cis}\ \cancel{-21.801}\ 158.199^\circ$

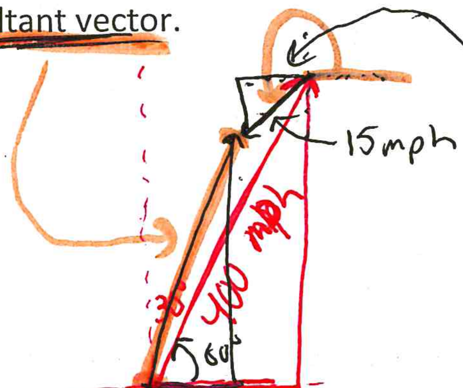
Plug the numbers into their corresponding places in trigonometric form. Use the i and j format or the < , > format (whichever you prefer).

$\langle \sqrt{29}\cos 158.199^\circ,\ \sqrt{29}\sin 158.199^\circ \rangle$

Where vectors become very useful is when you need to determine the result of multiple forces. Each force can represent its own vector and in component form, we know how much each vector is pulling left/right and up/down:

34. Imagine an airplane heads at a 30 degree bearing at 400 miles per hour (this is one vector), and the wind is blow southwest at 15 miles per hour (this is the second vector).

a) Sketch the first vector. Then sketch the beginning of the second vector at the end of the first vector. Finally draw in a third vector that begins at the start of the first vector and ends at the end of the second vector. This third vector is the result of the two forces and is therefore called the resultant vector.



b) Determine how much left/right and up/down for the first and second vector (put them in component form):

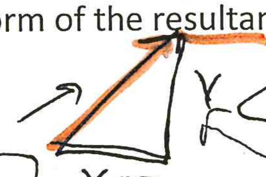
400 cis 60° $\quad\quad$ 15 cis 225°

$\langle 200, 200\sqrt{3} \rangle \quad\quad \left\langle -\frac{15\sqrt{2}}{2}, -\frac{15\sqrt{2}}{2} \right\rangle$

c) The component form of the resultant vector is found by combining (**adding**) the others vectors when they are in component form:

$$\begin{array}{r} \langle 200, 200\sqrt{3} \rangle \\ + \left\langle \frac{-15\sqrt{2}}{2}, \frac{-15\sqrt{2}}{2} \right\rangle \\ \hline \left\langle 200 - \frac{15\sqrt{2}}{2}, 200\sqrt{3} - \frac{15\sqrt{2}}{2} \right\rangle \end{array}$$

d) Finally, with component form of the resultant vector we can find its directional angle and magnitude:



$\langle 189.393, 335.804 \rangle$

$\|u\| = \sqrt{x^2 + y^2} = \boxed{385.531 \text{ mph}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \boxed{60.577^\circ}$